# Concept and importance of Magnetic circuits

Objectives :

To study Magnetic circuit, Magnetic field and operation

Chapters sections to be studied from the text book : 14.1,14.2

No. of Lecture = 03

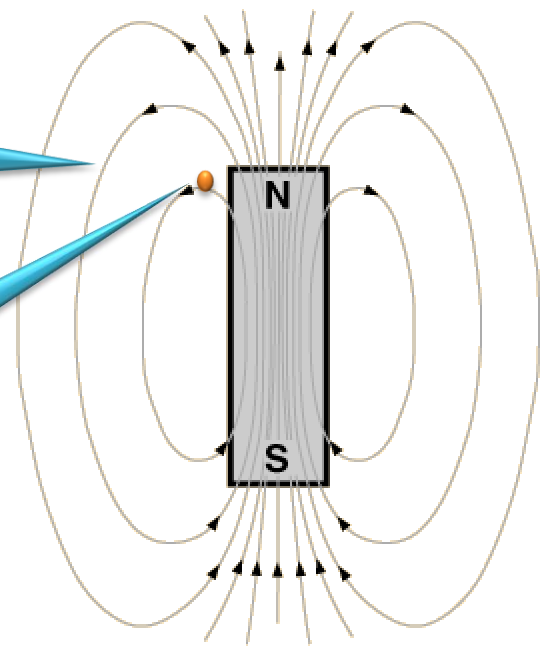
# Magnetic fields

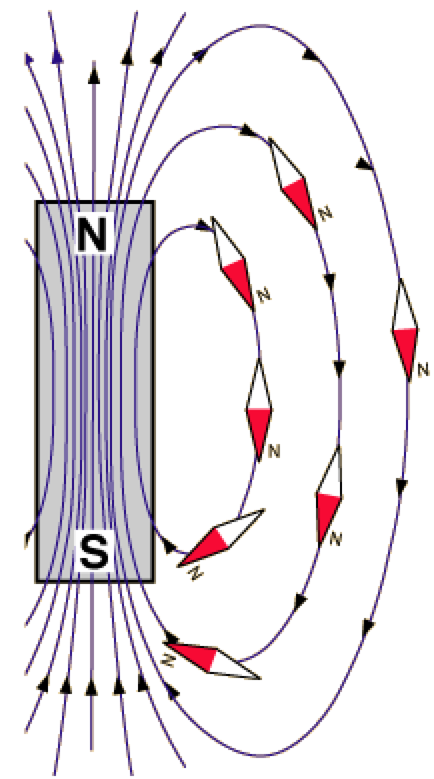
## Permanent Magnets

- Made of steel or iron alloys
- This exerts force on other magnet and on moving electric charge
- There is a magnetic field around a magnet.

Lines of magnetic force or Magnetic flux

Starts from the north pole and ends at south pole




Lines of magnetic flux forms a closed loop

# Magnetic fields

- The total flux passing through an area A is denoted by Φ, with an unit **Weber. (Wb)**

- When the Flux is uniformly distributed over an area A, then we call its as Magnetic flux density. $B = \Phi/A$ **tesla (T)**

  **1 tesla= 1 weber per square meter ( 1T= 1 W/m$^2$)**

- Flux lines nearer the magnet are more closely spaced. Flux has direction, So does flux density. $|B|=B$ (This is considered as a vector, which has direction and magnitude)

- Strength of the magnetic filed decreases with distance. (Magnitude of B decreases with the distance.

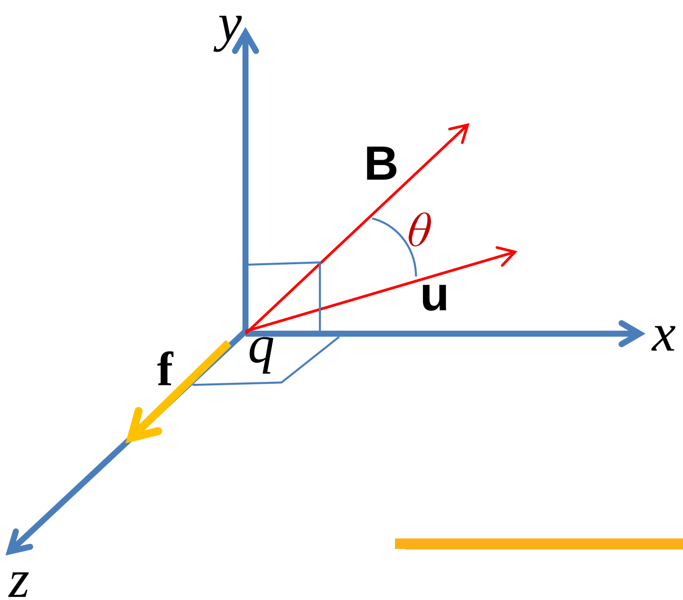
# Magnetic fields

We know that magnetic field exerts a force on moving electric charge, let the charge be $q$ Coulombs and velocity be $\mathbf{u}$ (|**u**|=u meter / second)

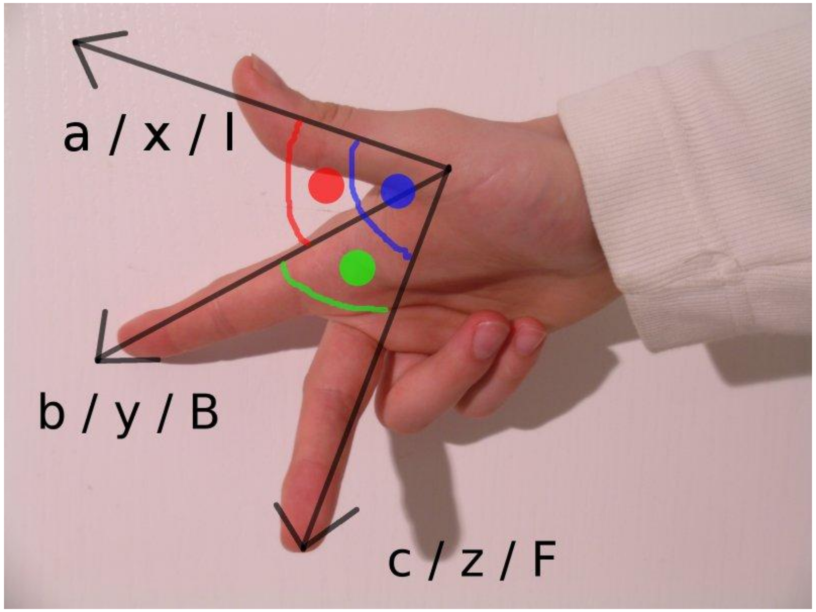
Suppose $q$ has magnetic flux density B, the force exerted on $q$ is **f** (vector with magnitude and direction)

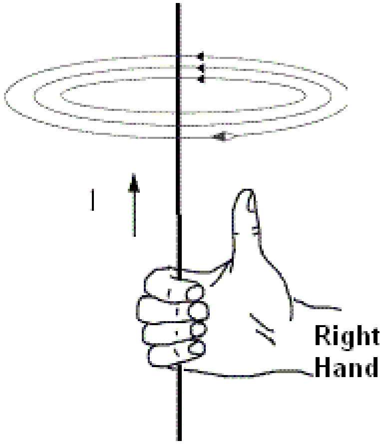
$$|\boldsymbol{f}|=f=quB\sin\theta$$

Where θ is the angle between **u** and **B**




- Vector f is normal to the plane containing u and B
- Direction of f is obtained through right-hand screw rule

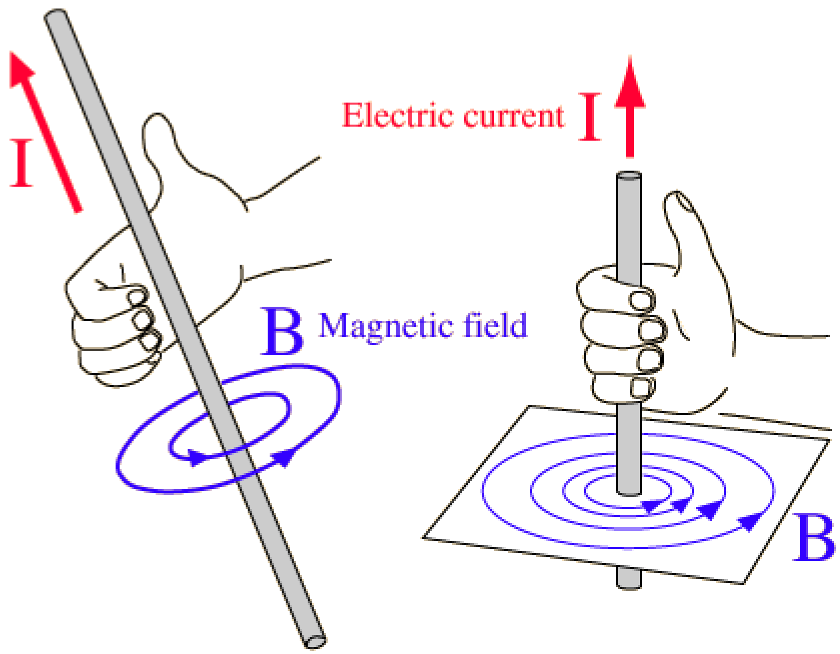



Magnetic field "curls" in direction around the wire as shown

# Currents and Magnetic fields

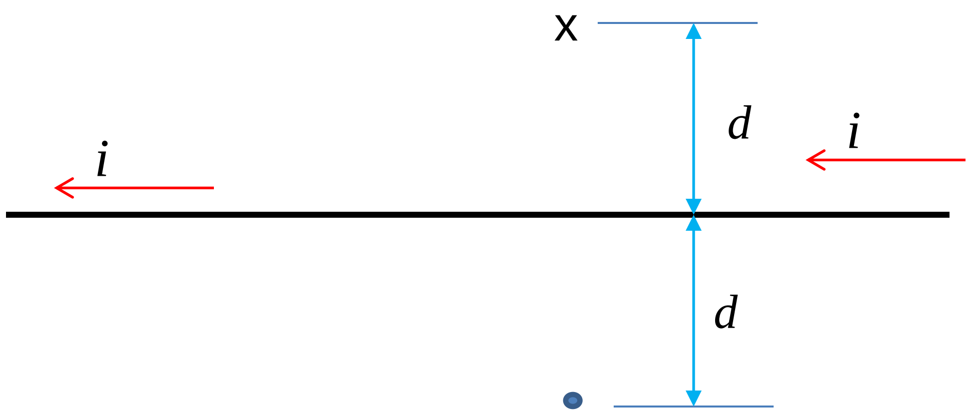
Current going through a conductor also gives rise to a magnetic field.




Consider a current I flowing through a long wire. Flux form a concentric circles around the conductor and the direction of the magnetic field is given by right hand rule.

# Currents and Magnetic fields

Consider two dimensional view



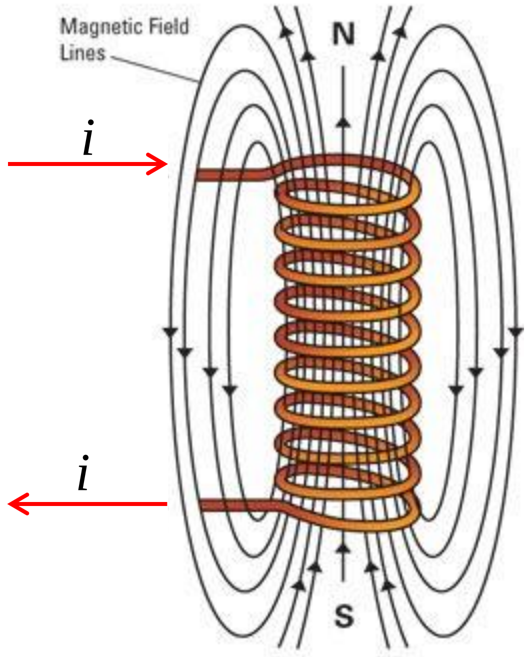

The magnitude of the flux density B at a point whose distance is d (measured perpendicularly) from the wire is given by

$$B = \frac{\mu}{2\pi d} i$$

Where μ is the property of the material , that surround the conductor and is called permeability of the material. **(Wb/A-m)**

# Currents and Magnetic fields

Similar magnetic field produced by winding a current carrying wire into a coil having N turns




Right hand rule can be restated now

***If the fingers on the right hand curl in the direction of the current then the thumb points in the direction of the magnetic field inside the core***

This is cylindrical coil is also called as **Solenoid**

If the radius of the core is $r$, and length of the cylinder is $l$, then, when $l >>>> r$, Flux density of the interior of the core is approximated by

$$B = \frac{\mu N}{l} i$$

# Currents and Magnetic fields

# Currents and Magnetic fields

Since the cross-sectional area of the of the core is $A=\pi r^2$, the total flux is given by

$$\phi = BA = \frac{\mu N \pi r^2}{l} i$$

From the above equation we can see that the flux in the core of the solenoid depends on the permeability $\mu$ of the core material.

Permeability of the free space= $\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m

Relative permeability $\mu_r = \mu / \mu_0$

For air and copper $\mu_r = 1$ and

For ferromagnetic material such as (iron, nickel, steel and cobalt) $\mu_r = 1000$ or greater.

# Currents and Magnetic fields

**Example:**

*For a solenoid , suppose the core has a radius of 0.01 m = 1cm and length of 0.2 m (20 cm). Determine the no. of turns required for a current of 1A to produce a magnetic flux density of 0.1 T in the core when the core material is a) air b) iron having relative permeability of 1200.*

$$\textbf{\textit{We know that}}\ \ N = \frac{Bl}{\mu i}$$

$$\textbf{\textit{a) For an air core,}}\ \mu = \mu_0 = 4\pi \,\mathbf{x}\, 10^{-7}\ Wb/A-m$$

$$N = \frac{(0.1)(0.2)}{(4\pi \,\mathbf{x}\, 10^{-7})} = 15{,}900\ \textit{turns}$$

# Currents and Magnetic fields

$$a)\ For\ an\ air\ core,\ \mu = \mu_r\,\mu_0 = 4\pi \,\mathrm{x}10^{-7}\,Wb/A-m$$

$$N = \frac{(0.1)(0.2)}{(1200)(4\pi\,\mathrm{x}10^{-7})} = 13.3\ turns$$

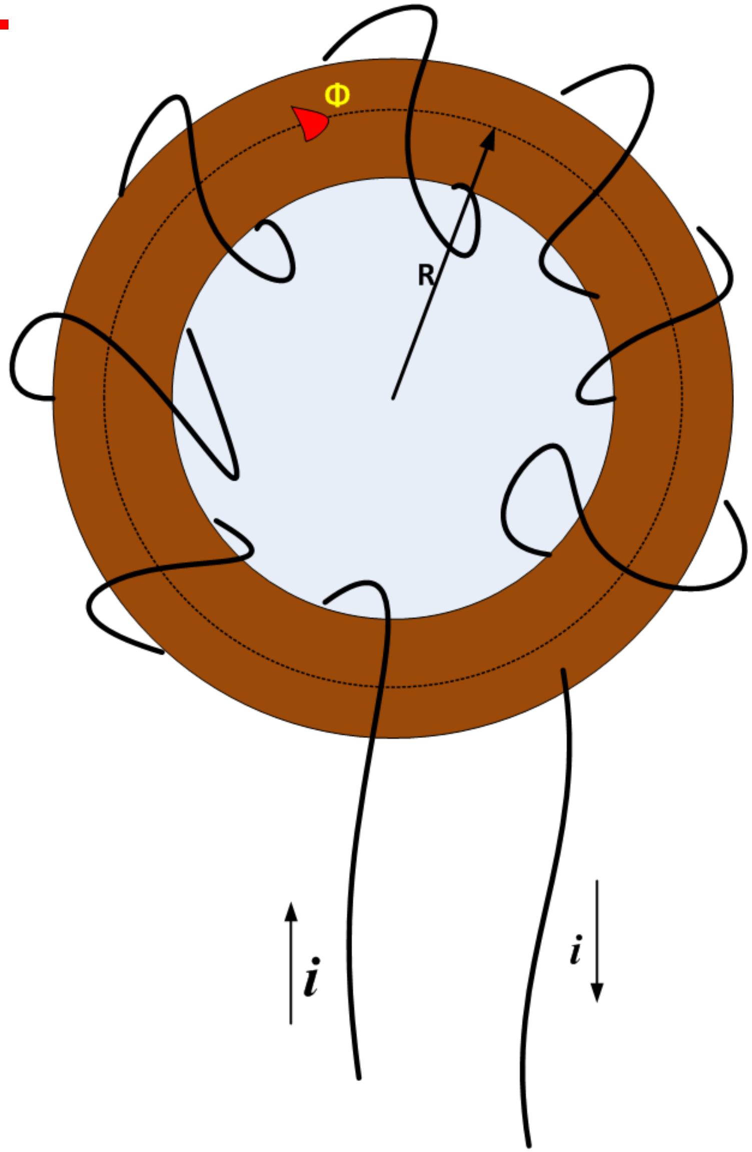
Exercise:

***For a solenoid , suppose that the core has a radius of 0.01 m and length of 0.2 m. Find the magnetic flux in the core with 100 turns when the current through the coil is 0.1 A and the core material is a) air b) iron core having relative permeability of 1500.***

Answer : a) $1.97 \times 10^{-7}$ Wb , b) $2.96 \times 10^{-4}$ Wb

# Toroid coil

- Doughnut shaped core

# Toroid coil

For the indicated current , the direction of the magnetic flux in the core is clockwise. Expression for the flux ϕ, is

$$\phi = BA = \frac{\mu N \pi r^2}{l} i$$

Where r is the cross-sectional radius and l is the average length of the core since

$$\boldsymbol{l = 2\pi R \quad , then \;\; \phi = \frac{\mu N r^2}{2R} i \quad and}$$

$$\boldsymbol{magnetic \;\; flux\; density\;\; is}$$

$$\boldsymbol{B = \frac{\phi}{A} = \frac{\mu N}{2\pi R} i}$$

# Toroid coil

- A toroid coil consist of 500 turns wound on an iron core having relative permeability $\mu_r$ = 1500. average radius R=0.1m = 10 cm , and cross-sectional radius r=0.02m=2cm. Determine the current that yields a magnetic flux of magnitude B=0.5 T in the core.

$$\mu_r = \frac{\mu}{\mu_0}; \quad \mu = \mu_r \mu_0 = 1500(4\pi \; x \, 10^{-7})$$

$$= 6\pi \; x \, 10^{-4} \;\; Wb/Am$$

$$i = \frac{2\pi RB}{\mu N} = \frac{2\pi(0.1)(0.5)}{(6\pi \; x \, 10^{-4})(500)} = 0.333A$$

# Toroid coil

Exercise :

- ***For the toroid , suppose the iron core has an average radius of 0.1m , a cross-sectional radius of 0.02m, and a relative permeability of 1200. Determine the number of turns required to produce a flux of 1 mWb in the core when the coil current is 1A.***

Answer : 332

# Toroid coil

- In more General form we can write $\phi = BA = \dfrac{\mu NA}{l} i$
- For a given flux $\phi$ is proportional to the product of the number of coil turns $N$ and the current $i$ .
- This product is called magnetomotive force (mmf) ($\mathscr{F}$ )

$$\mathscr{F} = Ni \quad \textit{(ampere turns) A-t}$$

# Magnetic Circuits

We know that $\phi = BA = \dfrac{\mu NA}{l} i$

$$\mathscr{F} = Ni$$

$$Let \quad \phi = \frac{\mathfrak{F}}{\mathfrak{R}} \quad where\ \mathfrak{R} = \frac{1}{\mu A} = \text{Reluctance}$$

$$\mathfrak{F} = \mathfrak{R}\,\phi$$

**Unit for reluctance is ampere-turns per weber (A-t / Wb)**

# Magnetic Circuits

Example:

- *For a toroid coil consisting of 500 turns, let the relative permeability $\mu_r$ = 1500. average radius R=0.1m = 10 cm , and cross-sectional radius r=0.02m=2cm. What is the current required to establish a flux density B=0.5 T .*

$$\phi = BA = (0.5)(\pi)(0.02)^2 = 6.28\ x\ 10^{-4} Wb$$

*The reluctance of the path of the flux is*

$$\Re = \frac{1}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2} = \frac{2(0.1)}{(6\pi \text{ x} 10^{-4})(0.02)^2} = 2.65\text{x}10^5\ A - t / Wb$$

# Magnetic Circuits

Thus mmf is given by

$$\mathscr{F} = \Re\phi = (2.65\text{x}10^{5})(6.28\text{x}10^{-4}) = 166.5\ A - t$$

*and*

$$i = \frac{\mathscr{F}}{\text{N}} = \frac{166}{500} = 0.332\,A$$

# Magnetic Circuits

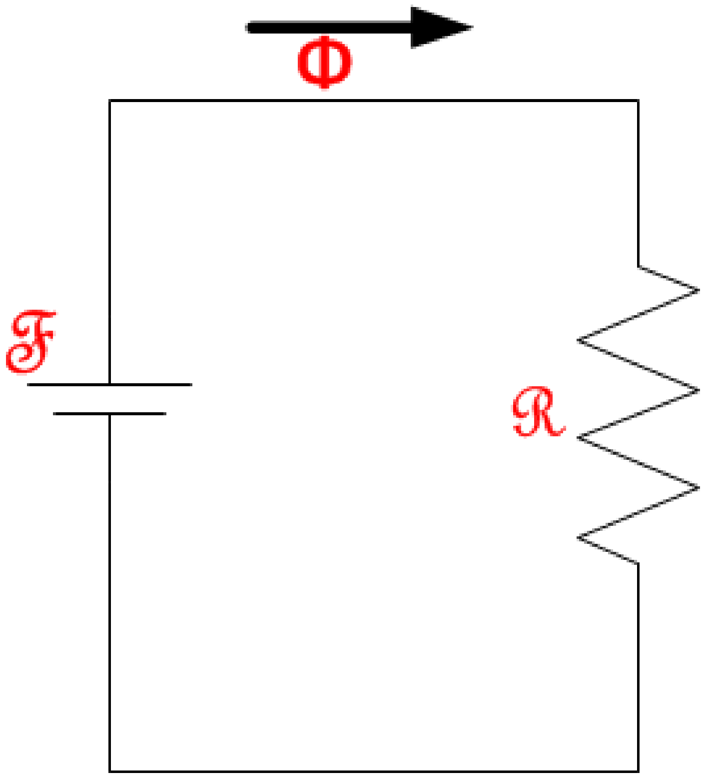
Example:

- ***For a toroid , suppose the iron core has an average radius of 0.1 m, a cross-sectional radius of 0.02 m, and a relative permeability of 1200. (a) Find the reluctance of the flux path. (b) Determine the mmf when the flux in the core is 1mWb. (c) What is the current in the coil for the case that it has 332 turns.***

***Answer : (a) 3.32 x $10^{-5}$ A-t/Wb, (b) 332 A-t (c) 1A***

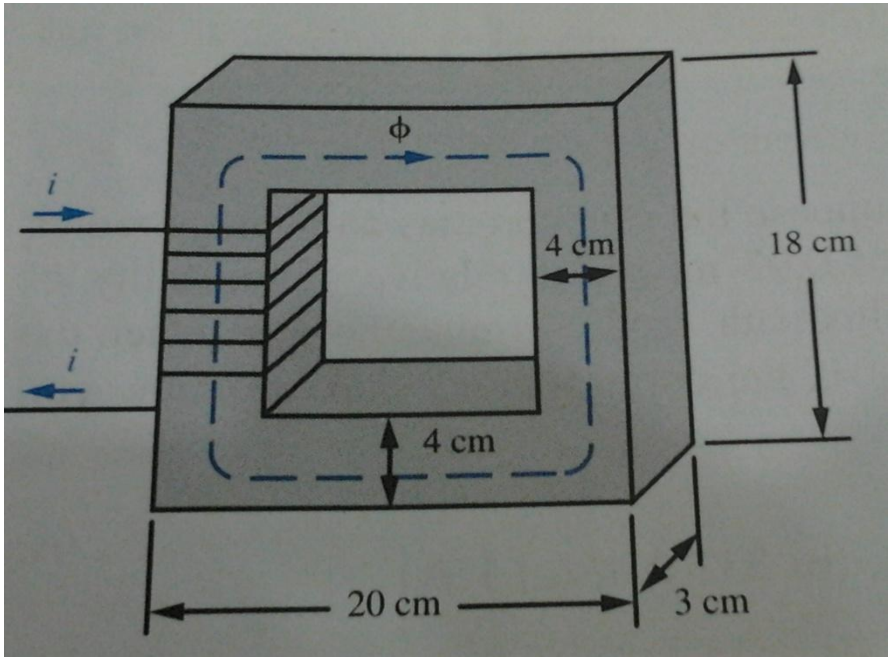
# Similarity between Magnetic and Electric circuits.

| Electric | Magnetic |
|---|---|
| Current $\boldsymbol{i}$ | Flux $\boldsymbol{\phi}$ |
| Voltage $\boldsymbol{v}$ | mmf $\mathscr{F}$ |
| Resistance **R** | Reluctance $\mathfrak{R}$ |
| V=Ri (ohms law) | $\mathscr{F}=\mathfrak{R}\boldsymbol{\phi}$ |



# Example:

- Consider a rectangular iron core, which has a relative permeability of $\mu_r$= 1500. Determine the reluctance and the magnetic flux in this core when a 200 turns coil has a current of 2A.



# Example:

From the diagram , we have.

Mean length of the flux path is $l$=16+14+16+14=60cm=0.6m

and cross-sectional area is A=(4 cm)(3 cm) =(0.03)(0.04)=0.0012m2=1.2 mm2, then the reluctance of the core is

$$\mathfrak{R}_{\mathcal{C}} = \frac{l}{\mu A} = \frac{0.6}{(1500)(4\pi \,\mathrm{x}\, 10^{-7})(1.2 \,\mathrm{x}\, 10^{-3})}$$

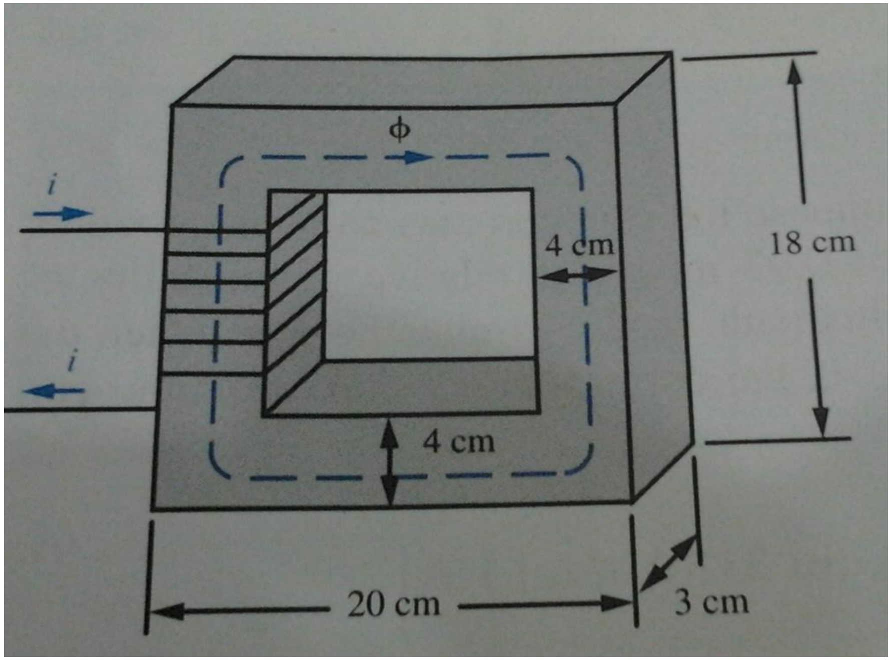
$$= 2.65 \,\mathrm{x}\, 10^{5} \; A - t / Wb$$

and the magnetic flux is

$$\phi = \frac{\mathfrak{F}}{\mathfrak{R}_{\mathcal{C}}} = \frac{Ni}{\mathfrak{R}_{\mathcal{C}}} = \frac{(200)(2)}{2.65 \, x \, 10^{5}} = 1.51 x 10^{-3} = 1.51 \; mWb$$

# Exercise

- For the rectangular iron core shown has a relative permeability of 1500. If the current in the coil is 1A, determine the number of coil turns needed to produce a magnetic flux of 3mWb in the core.




Answer : 795

# Magnetization Curve

Magnetic flux density B= ϕ/A and ϕ = $\mathscr{F}$ / $\Re$ , Reluctance $\Re$=1/ μ A, we can write

$$B = \frac{\mathscr{F}}{\Re A} = \frac{\mathscr{F}}{(1/\mu A)A} = \frac{\mu \mathscr{F}}{l}$$

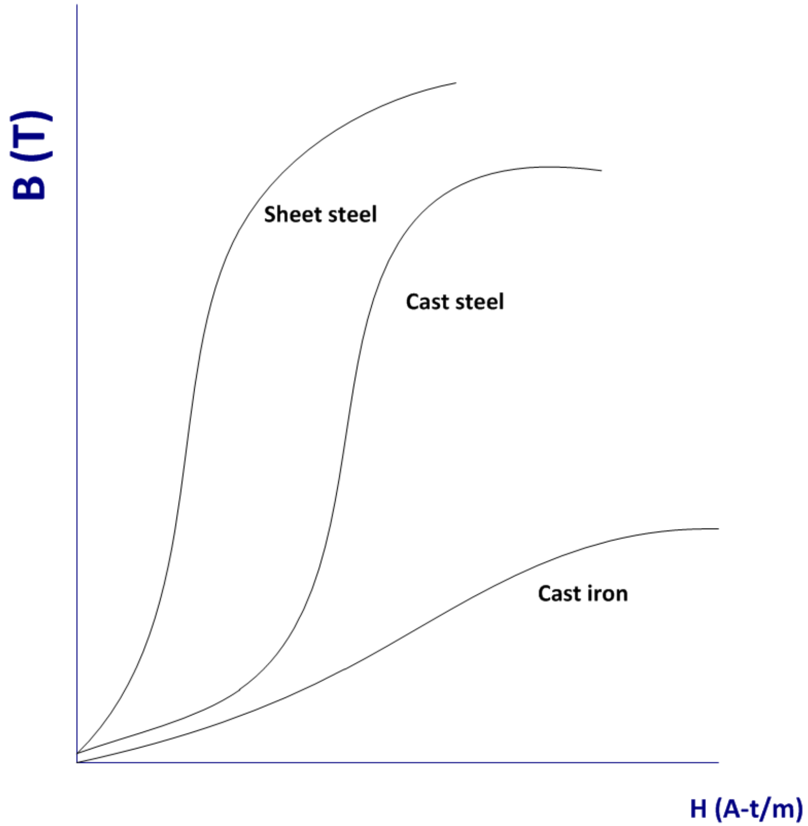
Thus we can write the above expression as B=μH, where

$$H = \frac{\mathscr{F}}{l} = \frac{Ni}{l}$$

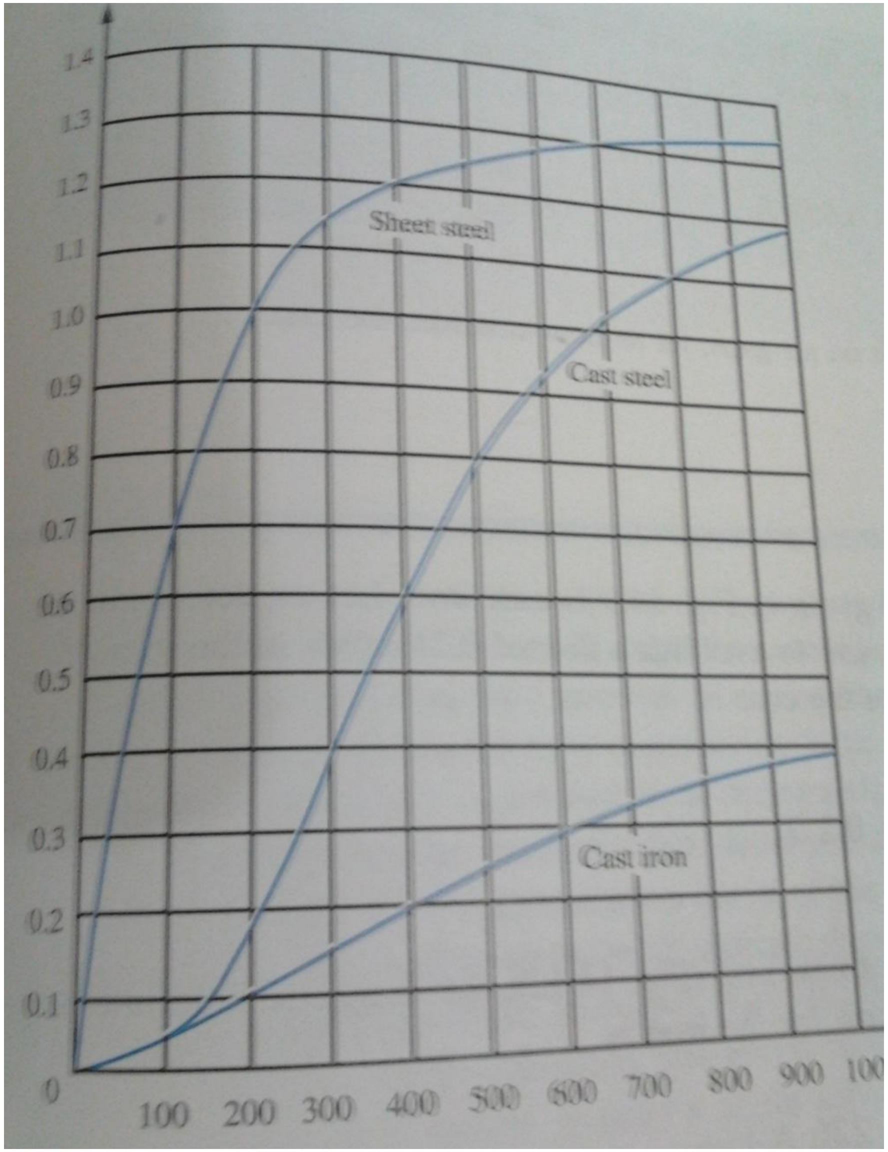
is the **magnetic field intensity** or **magnetization force**. (A-t/m) and is independent of core material.

# Magnetization Curve

- A plot of flux density B , versus magnetic field intensity H is called as magnetization curve or B-H curve.
- Graphically describes the average relationship between B and H.



# Magnetization Curve



# Magnetization Curve

Suppose toroid is constructed of sheet steel, and suppose magnetic flux density is to be B = 1T. Determine

(a) Relative permeability of this core

(b) the current required to produce the magnetic flux density when the average radius is 0.1m, the cross-sectional radius is 0.02m, and coil has 500 turns.

# Magnetization Curve

(a) From the magnetization curve, field intensity $H = 200$ A-t/m.

$$\mu = \frac{B}{H} = \frac{1}{200} = 5 \times 10^{-3} \text{ Wb/A-m}$$

Relative permeability

$$\mu_r = \frac{\mu}{\mu_0} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7}} = \underline{3980}$$

# Magnetization Curve

(b) mmf $\mathcal{F} = Hl = 200(2\pi)(0.1) = 1256$ A-t

Since $\mathcal{F} = Ni$, $i = \dfrac{\mathcal{F}}{N} = \dfrac{1256}{500}$

$$= 0.252 \text{ A}$$

# Magnetization Curve

**Exercise** :

- For a toroid constructed of cast steel is to have a magnetic flux density of 0.8 T

a) Determine the relative permeability of the core

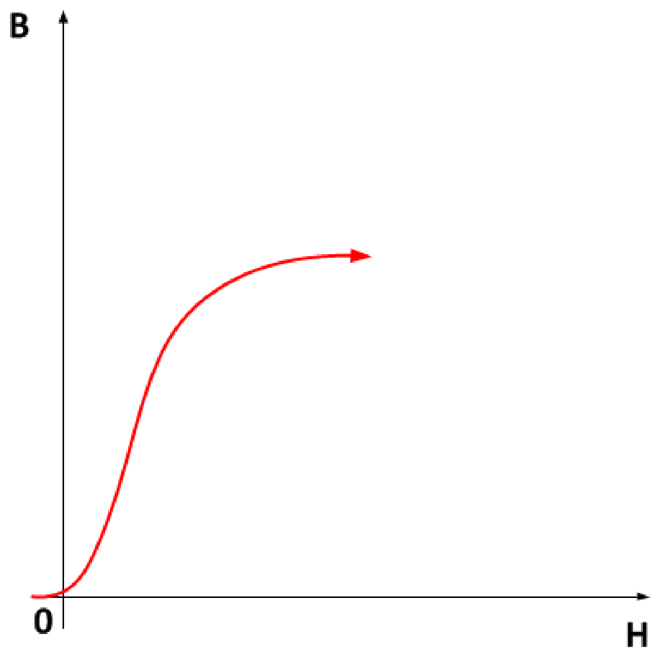
b) If the core has an average radius of 0.1 m and a cross-sectional radius of 0.02m, find the current required for a 500 turn coil.

Answer : a) 1273 b) 0.628A

# Hysteresis

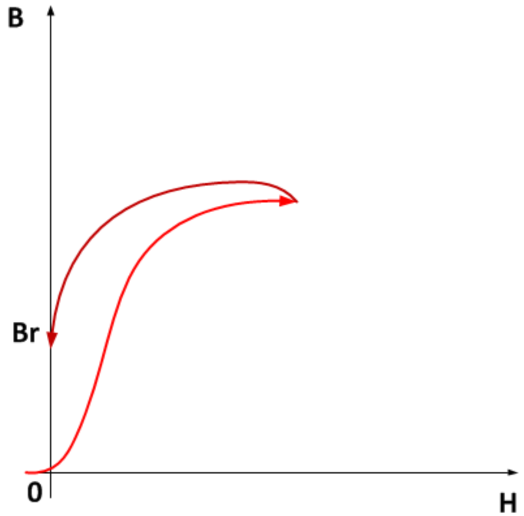
- What is demagnetized magnetic material ?

  It is one whose magnetic flux density is zero (B=0T) when no external magnetic force is applied. (H=0 A-t/m)
- When the magnetic force H is applied , the resulting flux density B is shown by the magnetization curve shown.



# Hysteresis

- For smaller H , B-H curve is relatively straight
- As H increases, curve bends and flattens out and further increase in H results in slight increase in B.
- Eventually further increase in H , yields no further increase in B.  Now the material is said to be saturated.
- Now at this moment , H is reduced to zero then B will decrease, but it will not take the original path.



# Hysteresis

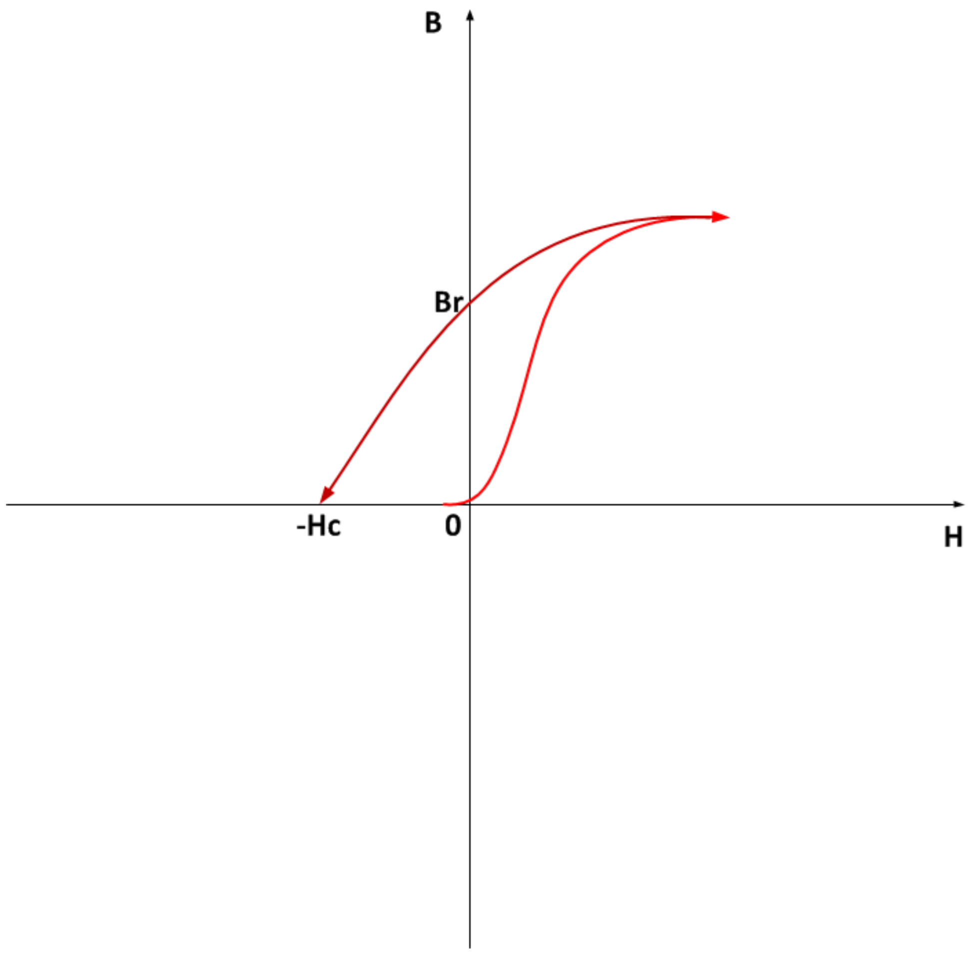
- The value obtained when H is reduced to zero is B=Br , which is the residual magnetism.

(Permanent magnets are made from materials, which has large residual magnetism)

- If H is increased once again , B will saturate and path is not the same as that of Br is reached.
- Suppose H is made increasingly negative, there will be place on H axis , say Hc, where flux density B=0.
- The value of Hc is called as Coercive force.

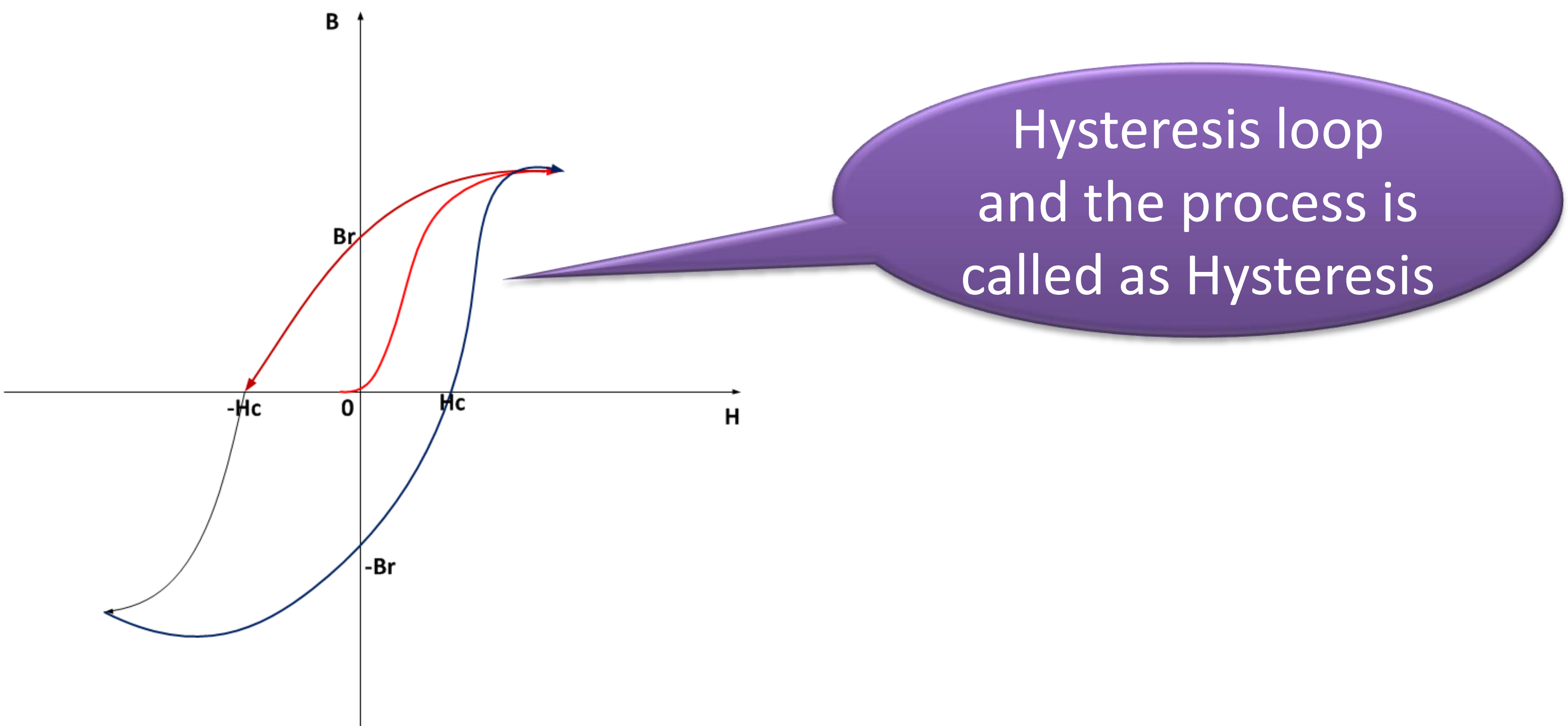
(Permanent magnets have large Coercive force)

# Hysteresis

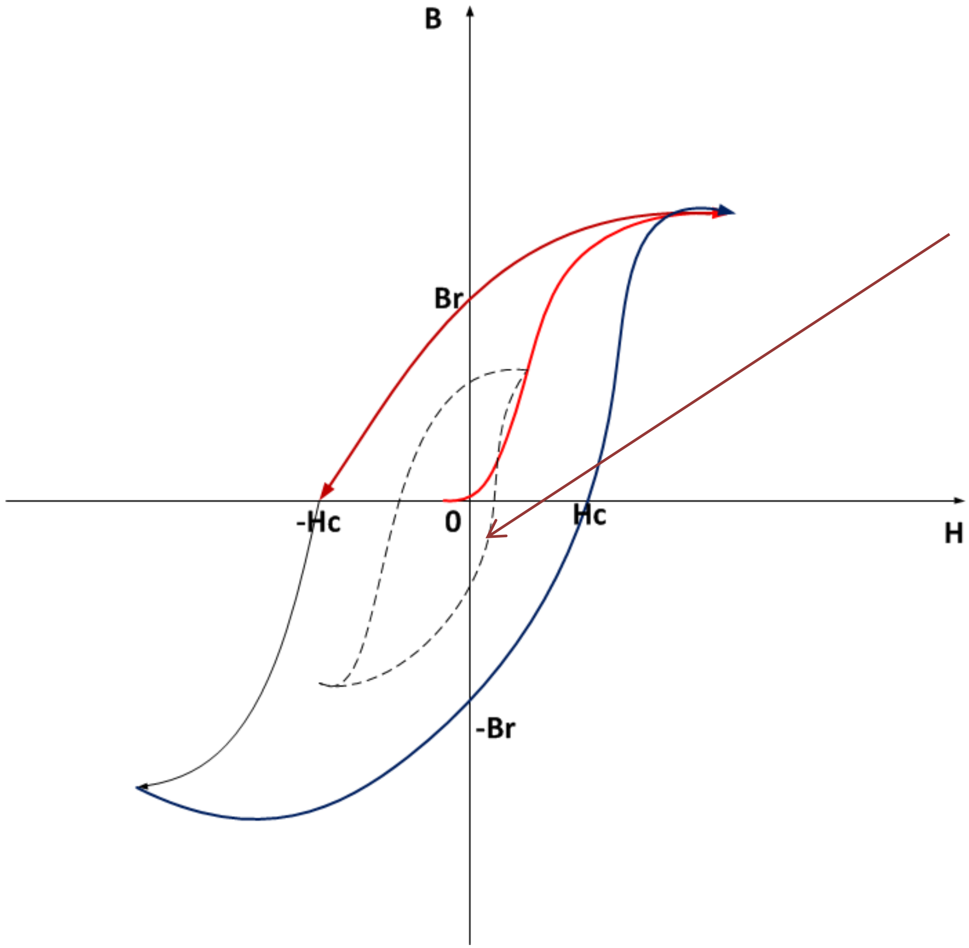


# Hysteresis

- If H is made still more negative , then it will saturate in opposite direction.
- Then , further increment in H will result in B to be saturated in the positive direction.



# Hysteresis



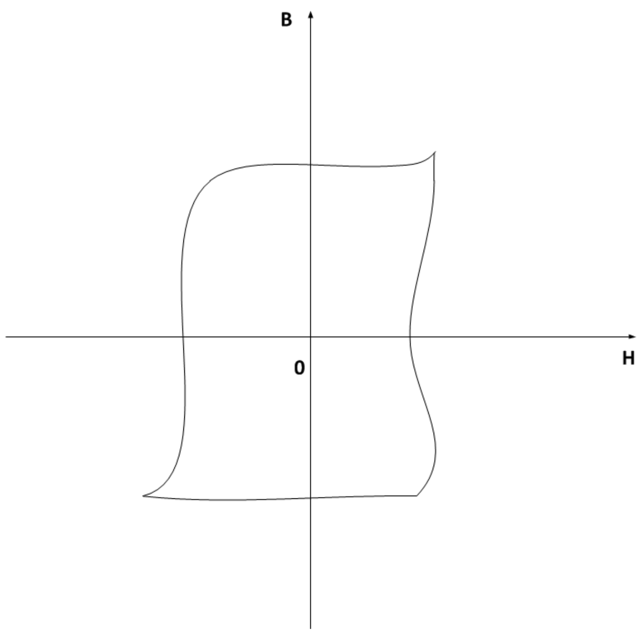

If the magnetic force applied to the demagnetized material is less than that required to produce saturation then the hysteresis loop is shown in the dashed line.

# Hysteresis

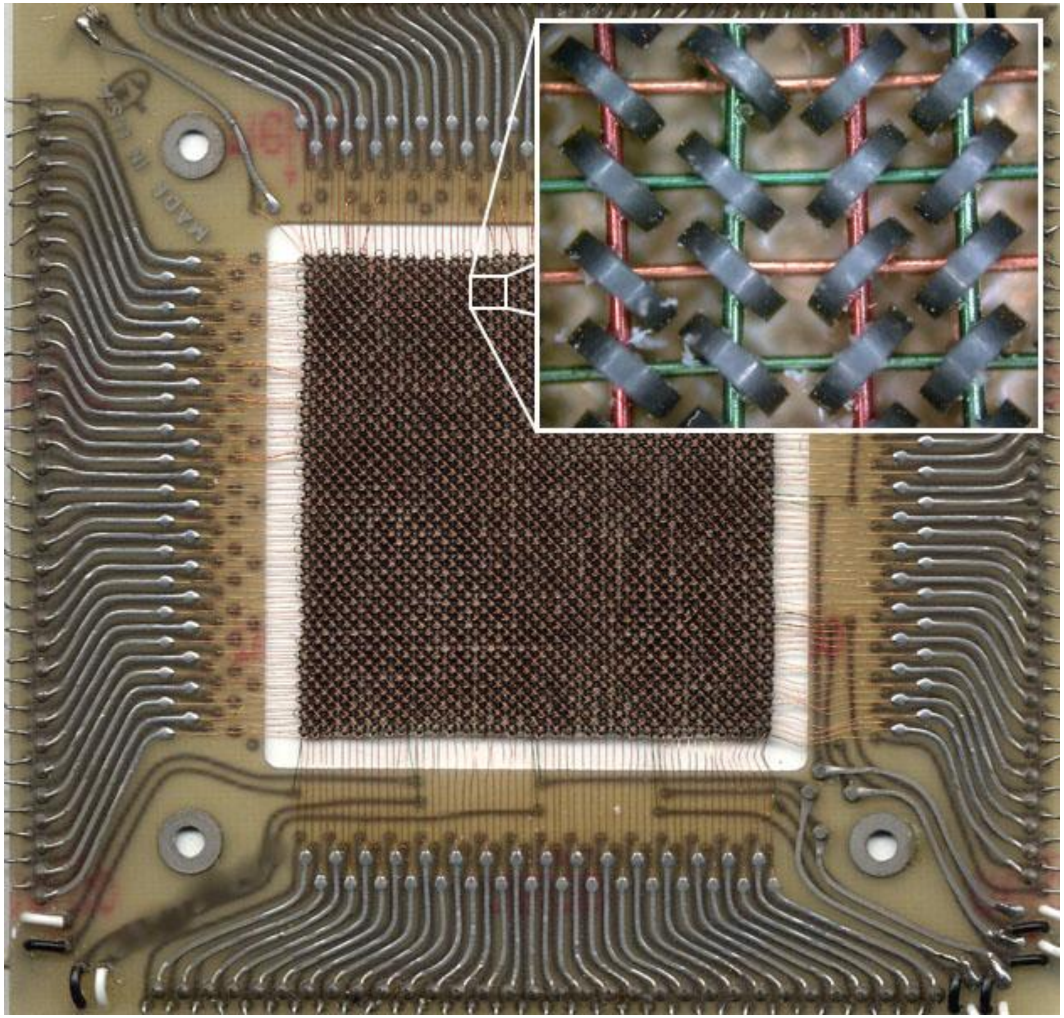
Some material have square shaped hysteresis loop, and called as square loop material, where the slope is very large.

Small change in H produces a large change in B.

Used in binary memory devices.



# Hysteresis



# Hysteresis

- When the magnetic materials are periodically magnetized and demagnetized using AC , power and energy is absorbed by the material and is converted to heat.
- Average hysteresis power loss $P_h = K_h f B_m^n$ , which is developed by Charles P. Steinmetz (1865-1923)

- Where $K_h$ is constant and $n$ (Steinmetz constant ) depend on the core material.
- $B_m$=maximum flux density
- $f$ is the frequency of the magnetic variation

# Transformers

- Electric energy can be efficiently transferred using transformers.
- Major use in power distribution.
- Step down and step up ac voltages and current.
- Electric power , which is generated typically from the voltage level 5 to 50 kV, can be stepped up in voltage and stepped down in current prior to distribution.
- Used in electronics, control, communication circuits and systems.

# Transformers

Magnetic induction (Michael Faraday 1831)

- Magnetic flux passing through a multiturn coil would induce a voltage , called an **electromotive force** (emf) across the coil, provided flux was time varying.
- Faraday’s Law says emf induced across the coil

$$e = N\frac{d\phi}{dt}\ \ Volts$$

$$where\ \ N = No.of\ turns\ and\ \phi\ is\ \ flux\ \ passes\ \ through$$

# Transformers

Let $\lambda = N\phi$ , call it as flux linkages. (Weber-turns), we can write $e = \dfrac{d\lambda}{dt}$

Let L be the inductance in coil of wire , we know voltage $\boldsymbol{v}$ across it and current $\boldsymbol{i}$ can be related as $\boldsymbol{v = L\dfrac{di}{dt}}$

Thus for a given inductor

$$\boldsymbol{L\frac{di}{dt} = \frac{d\lambda}{dt} \Rightarrow L\frac{di}{dt}\frac{dt}{di} = \frac{d\lambda}{dt}\frac{dt}{di}}$$

By the chain rule of calculus , $\boldsymbol{L = \dfrac{d\lambda}{dt} = N\dfrac{d\phi}{di}}$

# Transformers

We know that , $\mathscr{F} = Ni$ *(ampere turns) A-t*

and $\phi = \dfrac{\mathscr{F}}{\Re} = \dfrac{Ni}{\Re} = Ki$

Flux is directly proportional to the current.

Take the derivative of the above equation w.r.t $i$ both the side, we get

$$\frac{d\phi}{di} = K \;\; also \;\; K = \frac{\phi}{i}$$

Thus

$$\frac{d\phi}{di} = \frac{\phi}{i}$$

substituting this into $L = N\dfrac{d\phi}{di}$ , we get

# Transformers

$$L = \frac{N\phi}{i} = \frac{\lambda}{i}$$

Thus inductance is the flux linkages / ampere

1H=1 Wb/A

Substituting $\phi = \frac{Ni}{\Re}$ in above equation we get,

$$L = \frac{N(Ni/\Re)}{i} = \frac{N^2}{\Re}$$

- Inductance is directly proportional to the square of the number of turns.

# Transformers

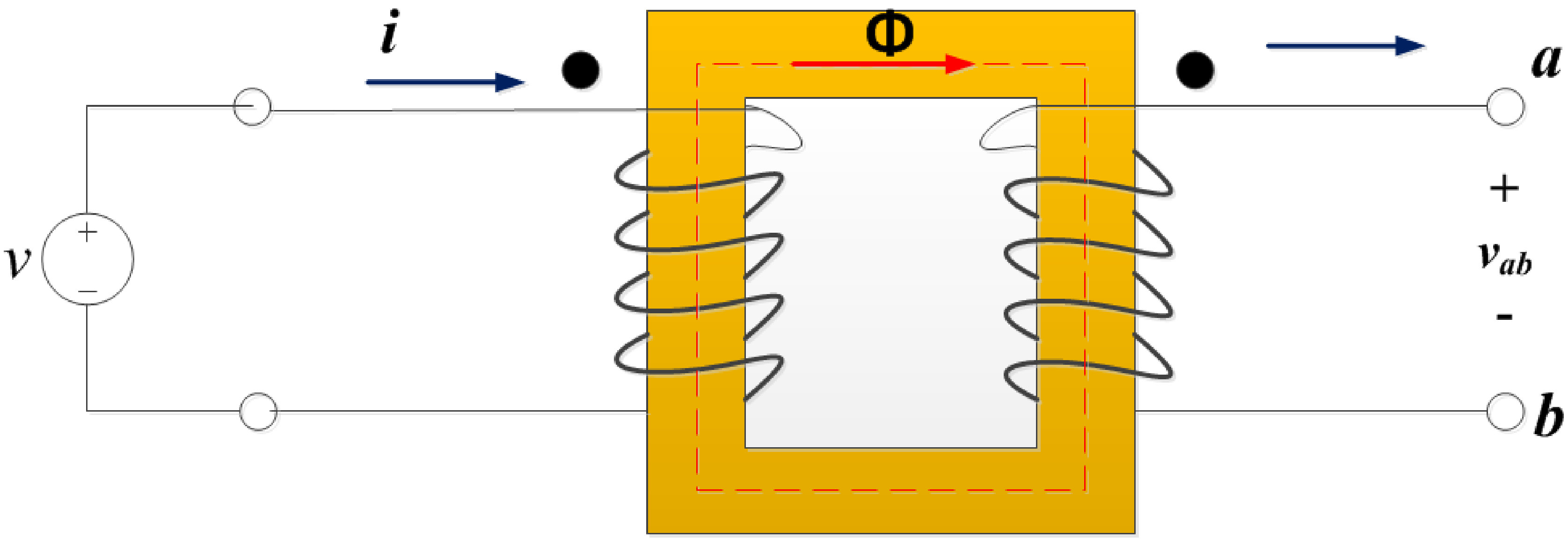
- Determine the inductance of a 500-turn coil wound on the toroidal core , when the core has a relative permeability of 1500, an average radius of 0.1m and crossectional radius of 0.02m.

# Transformers

## <u>Lenz's Law</u>

- Determines the polarity of the induced voltage.
- This Law states that, The polarity of the voltage induced by a changing flux tends to oppose the change in flux that produced the induced voltage.

Let us consider the core shown below



# Transformers

- Time varying voltage $v$ , will result in current $i$
- By the right hand rule, when $i$ is positive, then resulting magnetic flux is in the clockwise direction.
- When $i$ increases with time, ϕ also increases with time ($Ni = \mathscr{R}\phi$)
- The coil on the right would produce a counter clockwise flux that opposes the increasing flux ϕ.
- If nothing is connected across $a$ and $b$ , then there is no current , but there is voltage $v_{ab}$ induced across it.
- Polarity can be determined by assuming that a resistor is connected between $a$ and $b$
- When I is increasing with time , since $v=L\ di/dt$, v also a positive quantity

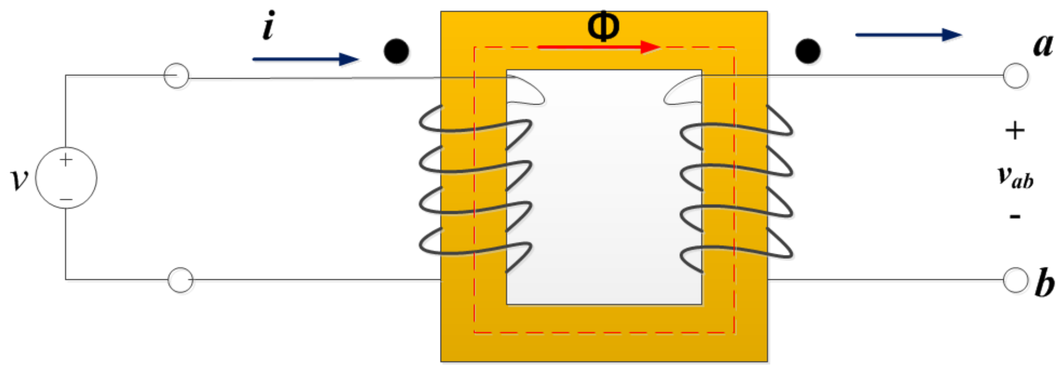
# Transformers

- If the $i$ is decreasing , then the coil on the right side tends to have a current that is opposite to the direction of the increasing current $i$, this reverses the polarity of the voltage.
- It can also be noted that , since $v = L\, di/dt$ , when $i$ is decreasing then v is a negative quantity.
- Polarity of the induced voltage also depends on how the coil on the right side is wound with respect to the left side.
- This is indicated with a dot (●), this indicates dotted ends are positive at the same time and negative at the same time.

# Transformers

**Magnetic Coupling**

Consider a rectangular core shown.




Let the coil on left side has $N_1$ turns and inductance $L_1$, and the right side coil has $N_2$ turns and inductance of $L_2$.

Time varying current *i* on the left side would induce a voltage across the coil on the right side and coils are said to be **magnetically coupled**.

# Transformers

When there is no load connected across $a$ and $b$,

$$\phi = \frac{N_1 i}{\Re}$$

When there is no flux leakage , induced voltage $v_{ab}$ is

$$v_{ab} = N_2 \frac{d\phi}{dt} = \frac{N_1 N_2}{\Re} \frac{di}{dt} = M \frac{di}{dt}$$

Where, $M = \dfrac{N_1 N_2}{\Re}$

The above relation is in the form $v = L\dfrac{di}{dt}$ , we can deduce M as an inductance and call it as **Mutual inductance** (Henry)

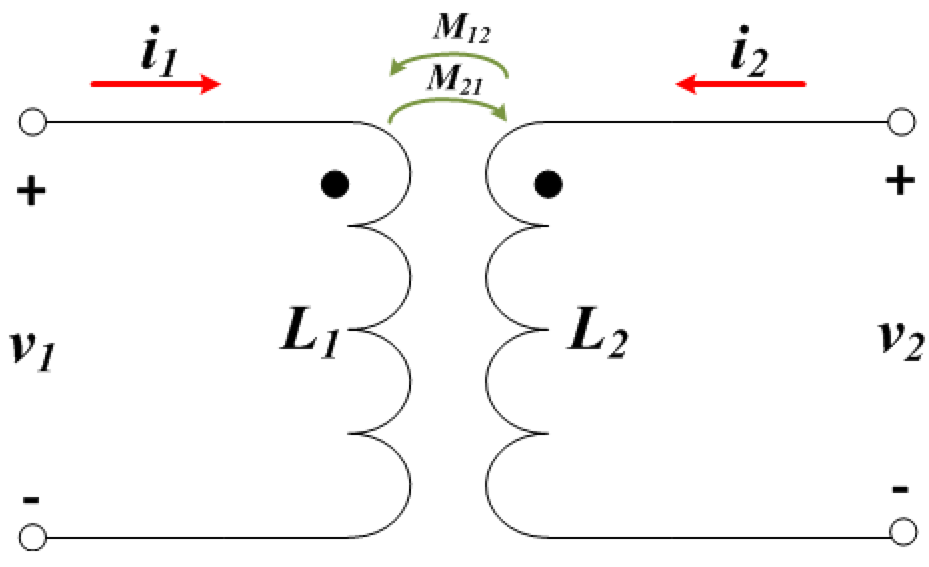
The inductances L1 and L2 called as **self inductance**.

# Transformers

- If $i$ is increasing $v_{ab}$ is positive and $i$ is decreasing $v_{ab}$ is negative.
- If the current $i$ goes into the dotted end of one coil , then for the other coil , the polarity of the induced voltage ($M\ di/dt$) is plus (+) at the dotted end.
- On the other hand, if current $i$ comes out of the dotted end of the one coil , then the polarity of the induced voltage ($M\ di/dt$) is (-).
- The pair of magnetically coupled coil is known as transformer , coil on the left side is primary and coil on the right side is secondary.

# Transformers

The circuit symbol of the transformer is




Connections may be from both windings, so $i_1$ and $i_2$ both are non-zero.

# Transformers

- Current $i_1$ going through the self inductance $L_1$ produces a voltage $L_1\, di_1/dt$. The current $i_2$ through secondary however, induces the voltage $M_{12}\, di_2/dt$ across the primary as well. By the principle of superposition.

$$v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

- Where $M_{12}$ is the mutual inductance corresponds to the voltage induced across the primary winding , due to the current in the secondary windings.

# Transformers

- We can also obtain $v_2 = M_{21}\dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt}$

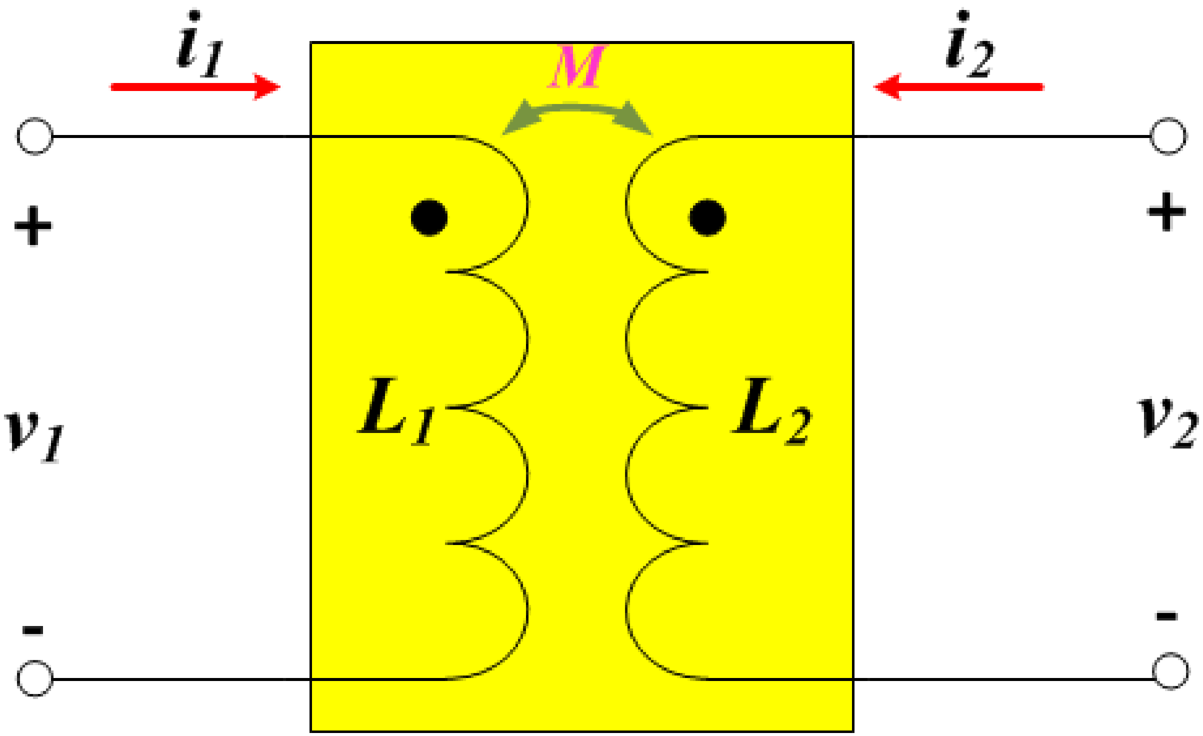
- Where $M_{21}$ is the Mutual inductance that corresponds to the voltage induced across the secondary , due to current in the primary.

- The relation suggests $M = \dfrac{N_1 N_2}{\Re} = M_{12} = M_{21}$

- Thus M can be the Mutual inductance of the transformer.

# Transformers

- With mutual inductance M , the transformer can be represented as follows



# Transformers

The equations for $v_1$ and $v_2$ can be written as

$$v_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \quad and$$

$$v_2 = M\frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Energy stored in the transformer is given by

$$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$

Use of transformer is limited to ac application only.