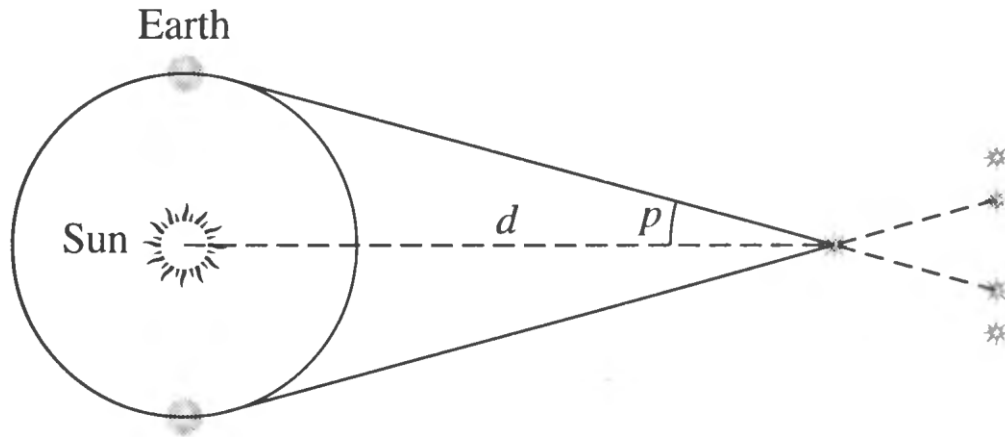


Chapter 3

The Continuous Spectrum of Light

**Stellar Parallax -
Distance Determination of Nearby Stars**



$$d = \frac{1 \text{ AU}}{\tan p} \approx \frac{1}{p} \text{ AU}$$

$$d = \frac{1}{p''} \text{ pc.}$$

where, 1 parsec (pc) = 206264.806 AU = 3.08×10^{16} m = 3.26 ly

A star with a parallax of 1'' is 1 pc away and even the closest star has a parallax of less than 1''!

Friedrich Wilhelm Bessel (1784 – 1846)

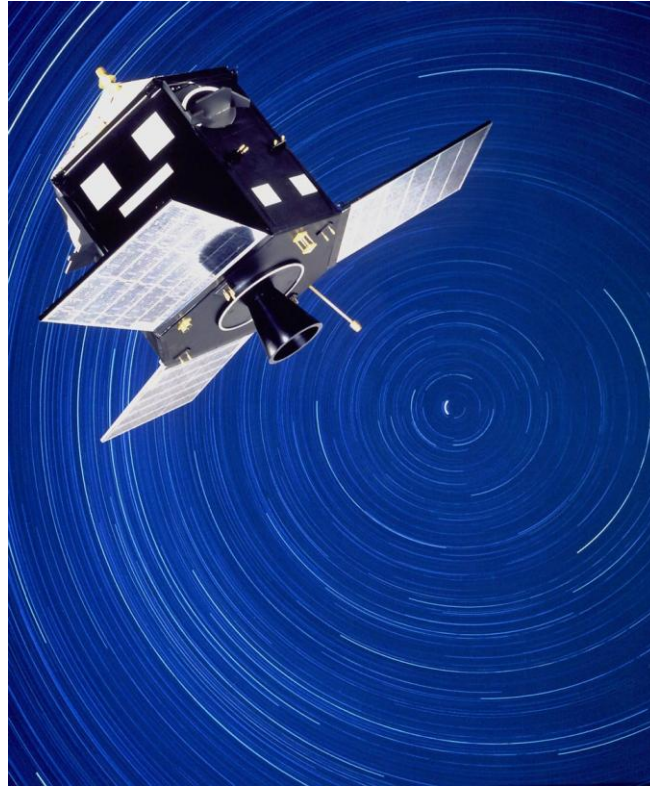
First person to be able to measure the parallax to 61 Cygni – 0.316"

Parallax of Proxima Centauri – 0.77"
(the largest among all stars with this base-line, 1 AU)



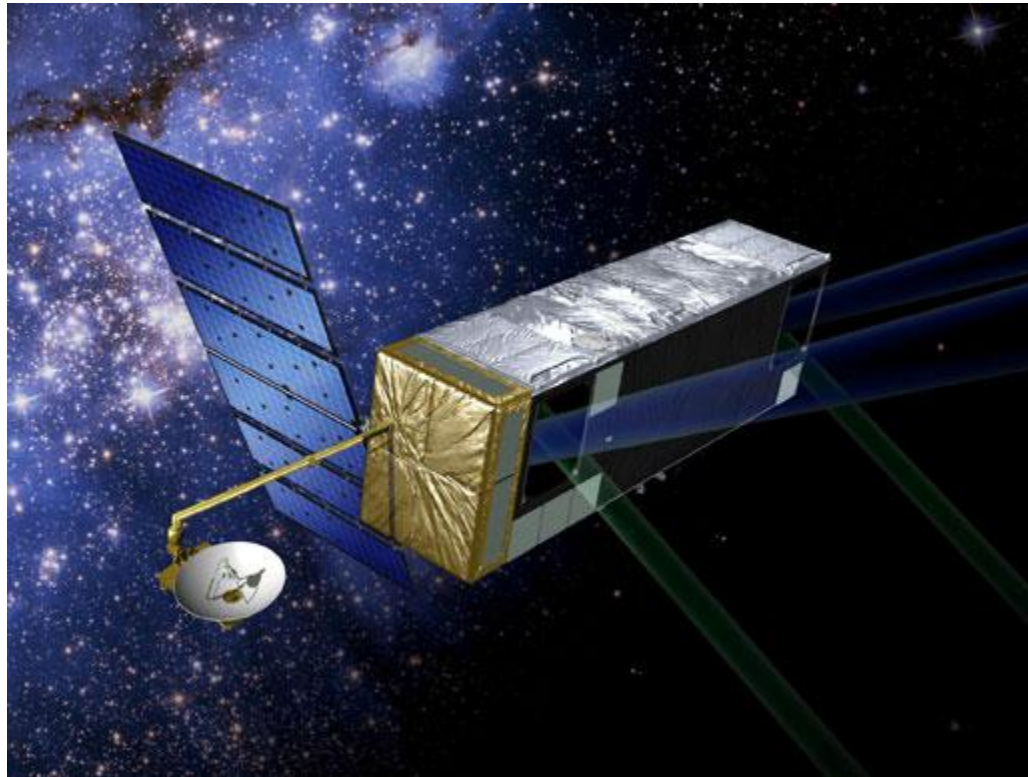
No wonder Tycho Brahe could not detect any parallax at all!

ESA's Hipparcos Space Astrometry Mission



*Measured parallax angles with accuracies $\sim 0.001''$,
i.e., distances up to 1 kpc.*

NASA's SIM PlanetQuest Mission



Will be able to determine parallax angles as small as 4 micro-arcsec: distances upto 250 kpc away!

Problem 3.5

- (a) The Hipparcos resolution of parallax measurement is $0.001''$. At what distance a dime (diameter - 1.9cm) need to be in order to observe an angle of $0.001''$?
- (b) Assume the grass grows at the rate of 5 cm per week.
- How much does grass grow in 1 sec?
 - How far from the grass you would need to be to see it grow at an angular rate of 4 micro-arcsec (resolution of SIM PlanetQuest Mission)?

Ans (a) 3920 km
(b) (ii) 4 km

Apparent Brightness

Brightness is the radiant flux, F , received from the star.

$$F = \frac{L}{4\pi r^2}$$

where L is the luminosity, the total energy that a star emits per second — Inverse Square Law

Problem 3.2

At what distance from a 100-W bulb is the radiant flux equal to the solar irradiance?

Ans: 7.63 cm

The Magnitude Scale

Five-magnitude difference equals a flux ratio of 100.

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$$

Magnitude scale is a reverse scale, smaller magnitude means higher flux!

$$m_1 - m_2 = -2.5 \log \left(\frac{F_1}{F_2} \right)$$

Absolute Magnitude and the Distance Modulus

Absolute magnitude of a star is the apparent magnitude of a star *if* it were located at 10 pc.

$$m - M = 5 \log(d) - 5$$

This is the distance modulus of the star and d is the distance to the star in pc.

Problem 3.3

The parallax angle for Sirius is $0.379''$.

(a) Find the distance to Sirius in parsec, light years, AU, and in meter.

(b) Determine the distance modulus of Sirius.

Ans (a) 2.64 pc

Ans (b) -2.89

Deriving Flux from Apparent Magnitude

$$M = M_{\text{Sun}} - 2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right)$$

where, $M_{\text{Sun}} = +4.74$, and $L_{\text{Sun}} = 3.8 \times 10^{26} \text{ W}$

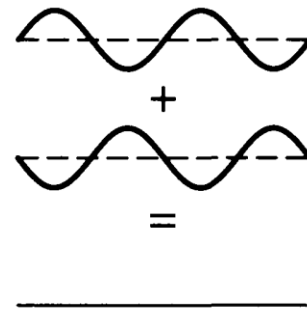
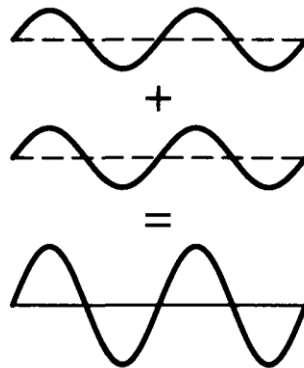
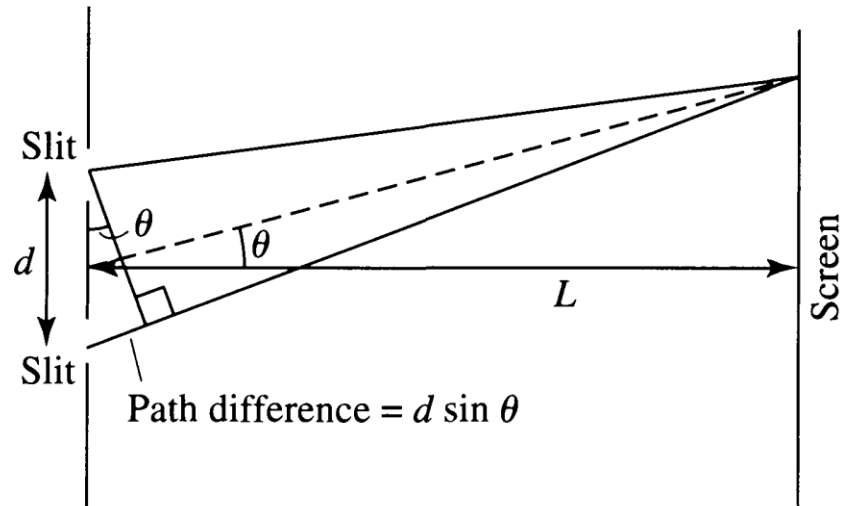
$$m = M_{\text{Sun}} - 2.5 \log_{10} \left(\frac{F}{F_{10,\odot}} \right)$$

where, $F_{10,\odot}$ is the flux of the Sun received *if* it were at 10 pc.

Assignment: Prob. 3.6

What is Light?

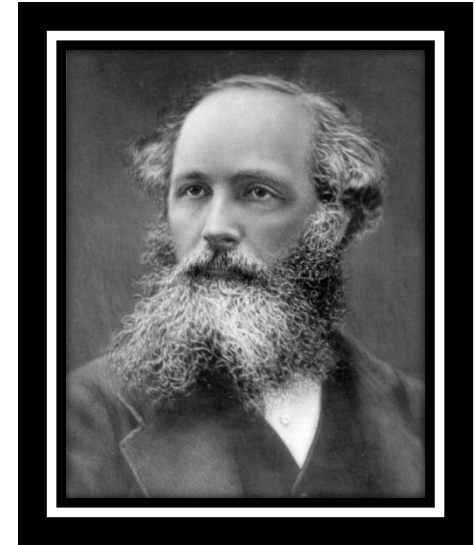
Young's Double Slit Experiment



If Light behaves like wave, what are these waves?

James Clerk Maxwell (1831 - 1879)

- (i) $\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$ (Gauss's law),
- (ii) $\nabla \cdot \mathbf{B} = 0$ (no name),
- (iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ (Faraday's law),
- (iv) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ (Ampère's law with Maxwell's correction).



Maxwell's equations predicted that EM waves travel through vacuum with the speed of light!

In 1889, Hertz found the experimental proof.

Heinrich Hertz (1857 - 1894)

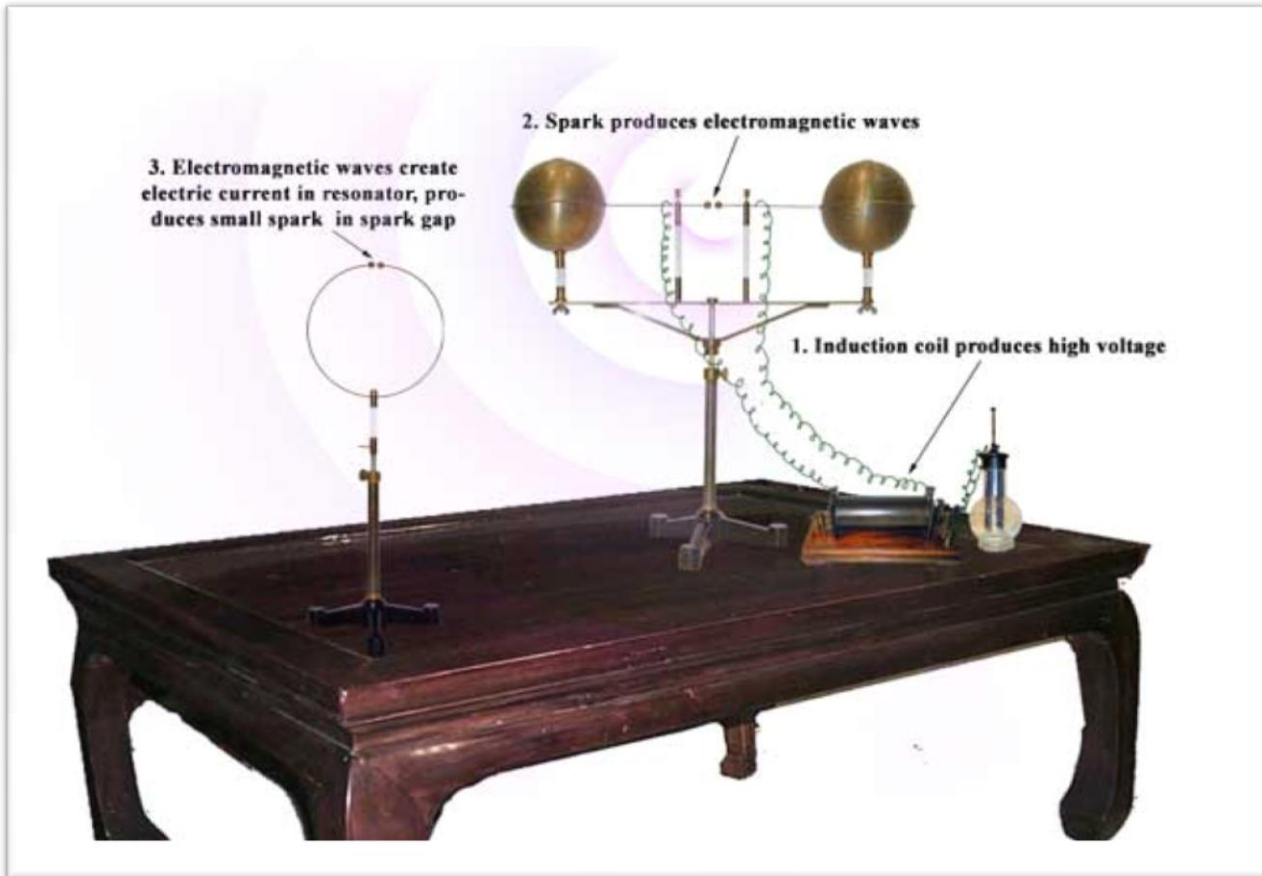
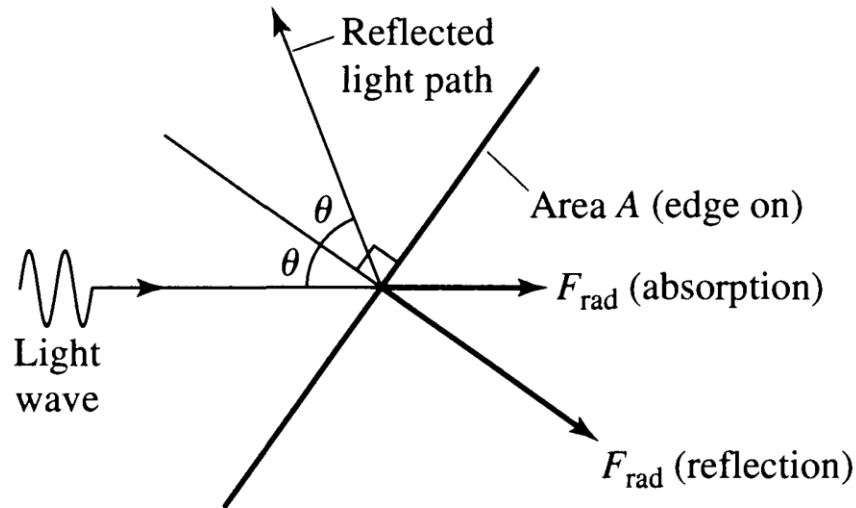


Image Source: Hertz Spark Museum

Radiation Pressure



Radiation Pressure
(Absorption)

$$F_{\text{rad}} = \frac{\langle S \rangle A}{c} \cos \theta$$

Radiation Pressure
(Reflection)

$$F_{\text{rad}} = \frac{2\langle S \rangle A}{c} \cos^2 \theta$$

Is it red-hot?

Relationship between Color & Temperature

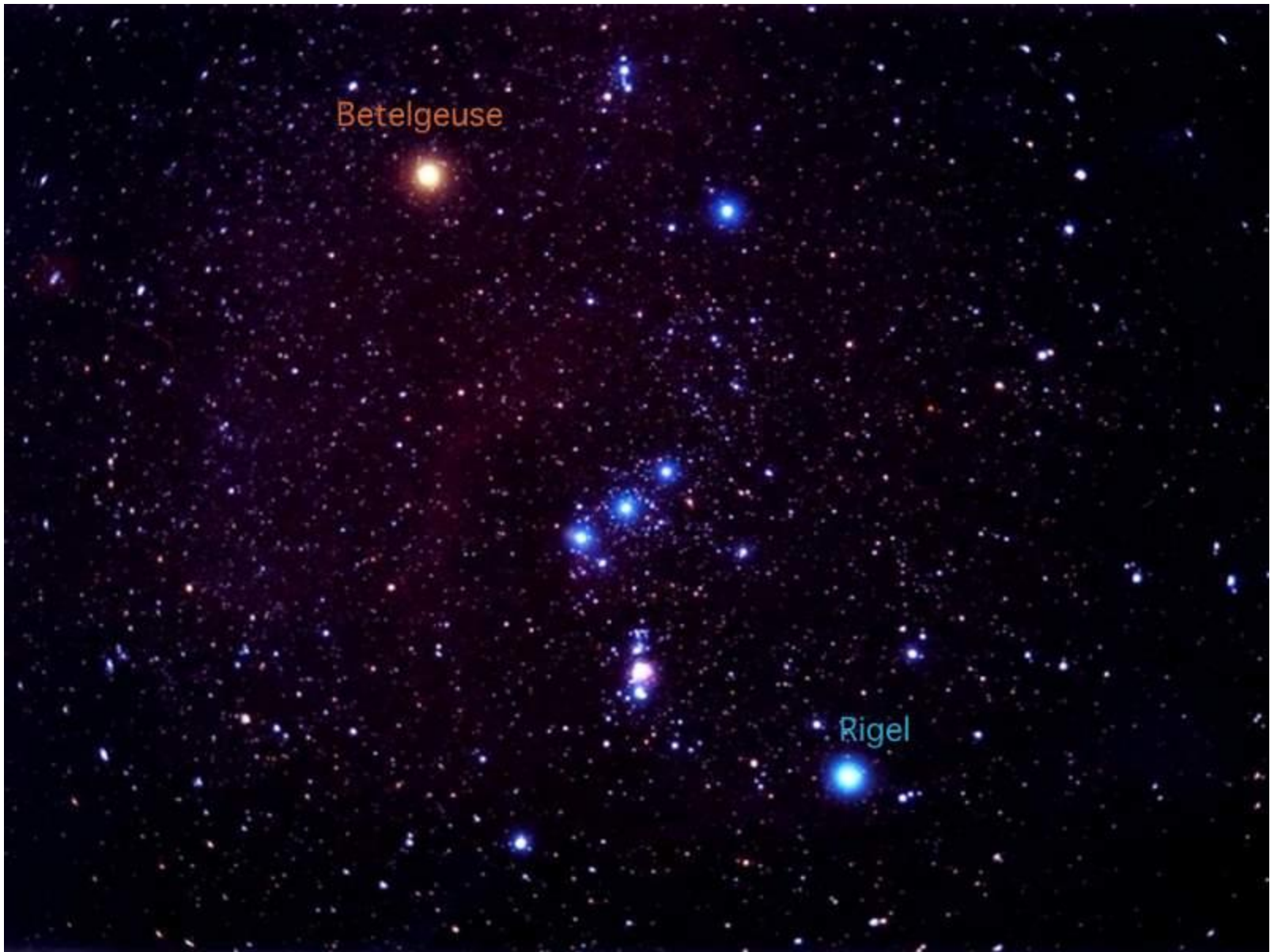
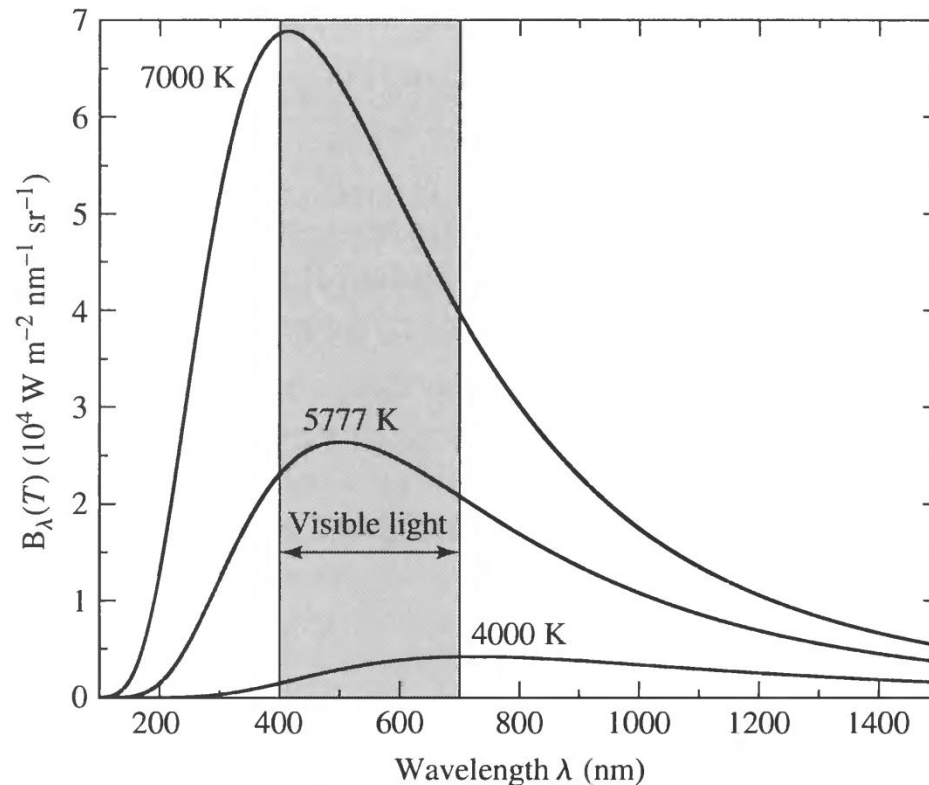


Image Source: <http://www.astronomyisrael.com/2011/01/>

Blackbody Radiation

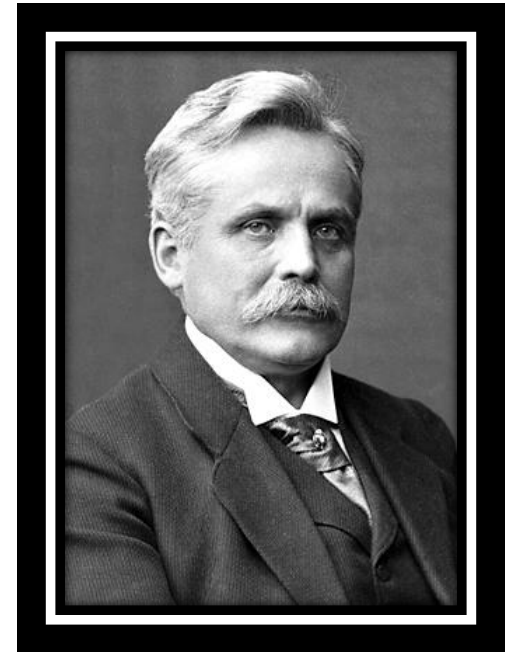
A blackbody is an object that absorbs all the radiation that strikes it. The radiation it emits is called blackbody radiation.



Wilhelm Carl Werner Otto Fritz Franz Wien (1864 – 1928)

Wien's Displacement Law:

$$\lambda_{\max} T = 0.002897755 \text{ m K}$$

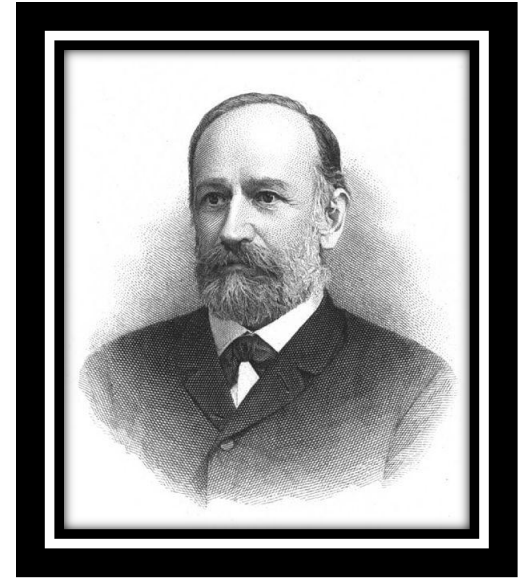


Betelgeuse has a surface temperature of 3600 K and Rigel has a surface temperature of 13,000 K.

The Stefan-Boltzmann Equation

In 1879, Stefan experimentally showed that the luminosity of a blackbody is related to its temperature and area by,

$$L = A\sigma T^4$$



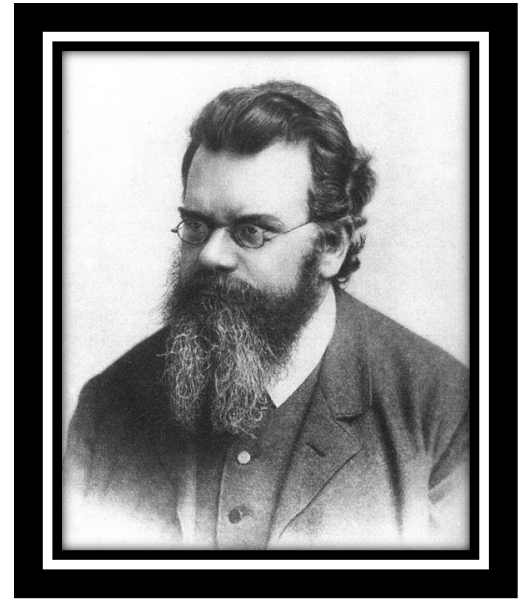
Joseph Stefan (1835 – 1893)

The Stefan-Boltzmann Equation

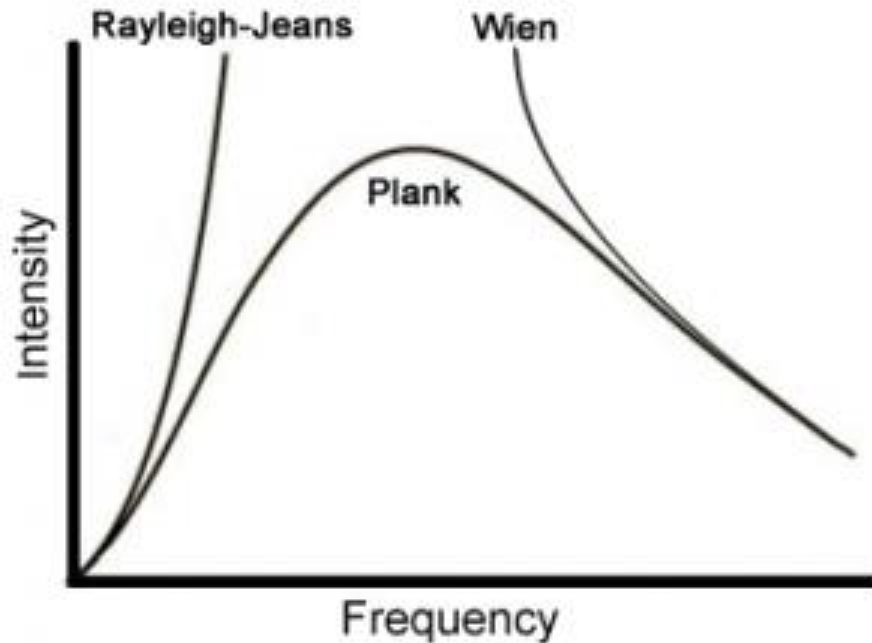
In 1884, Boltzmann theoretically derived this equation using Maxwell's expression for radiation pressure and the laws of thermodynamics.

$$L = A\sigma T^4$$

$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4}$$



Ludwig Boltzmann(1844 – 1906)



Rayleigh-Jeans Law

$$B_{\lambda}(T) \simeq \frac{2ckT}{\lambda^4}$$

Wien's Law

$$B_{\lambda}(T) \simeq a\lambda^{-5}e^{-b/\lambda T}$$

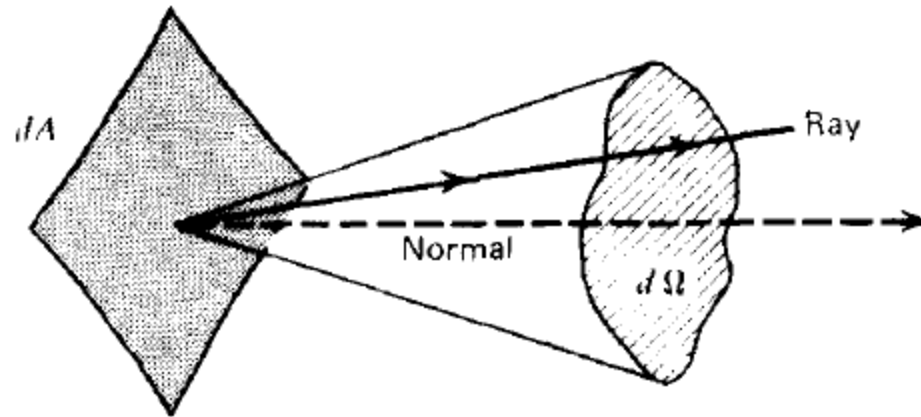
Planck's Law

$$B_{\lambda}(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

where $h = 6.62 \times 10^{-34} \text{ J s}$

Planck's Function & Astrophysics

Specific Intensity or Brightness



$$dE = I_{\lambda} dA dt d\Omega d\lambda$$

This is the energy crossing the area dA in time dt and in wavelength interval $d\lambda$. I_{λ} is the specific intensity or brightness.

If a star behaves like a blackbody, we can equate the specific intensity I_λ to Planck's Function and start from there...

Let us find the total energy emitted by a star every second in a given wavelength range – the monochromatic luminosity – by setting I_λ equal to the Planck's function and integrating,

$$L_\lambda d\lambda = \frac{8\pi^2 R^2 hc^2 / \lambda^5}{e^{hc/\lambda kT} - 1} d\lambda$$

And the Flux received at distance r ,

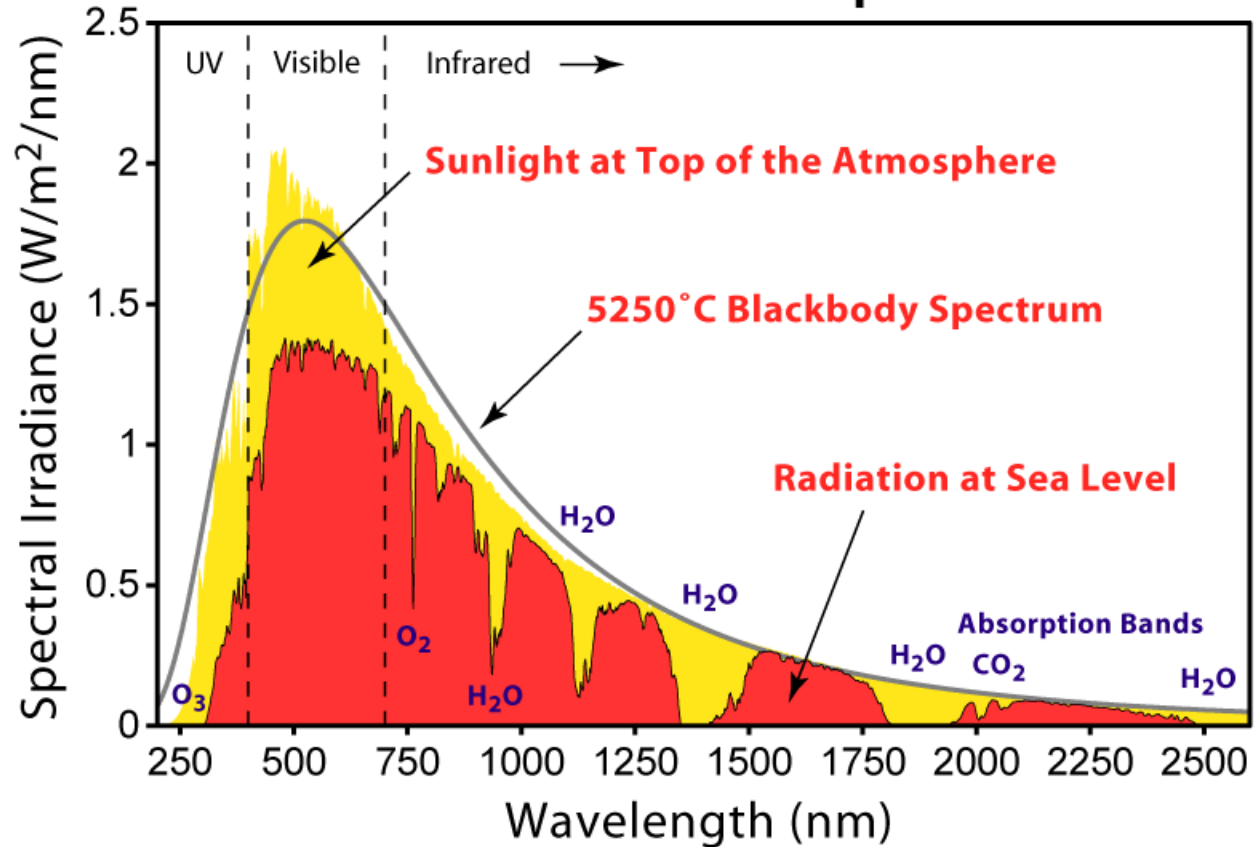
$$F_\lambda d\lambda = \frac{2\pi hc^2 / \lambda^5}{e^{hc/\lambda kT} - 1} \left(\frac{R}{r} \right)^2 d\lambda$$

Now integrating over all wavelengths, we get the total energy emitted by a star every second

$$\int_0^{\infty} B_{\lambda}(T) d\lambda = \frac{\sigma T^4}{\pi}$$

Assignment: Problem 3.14

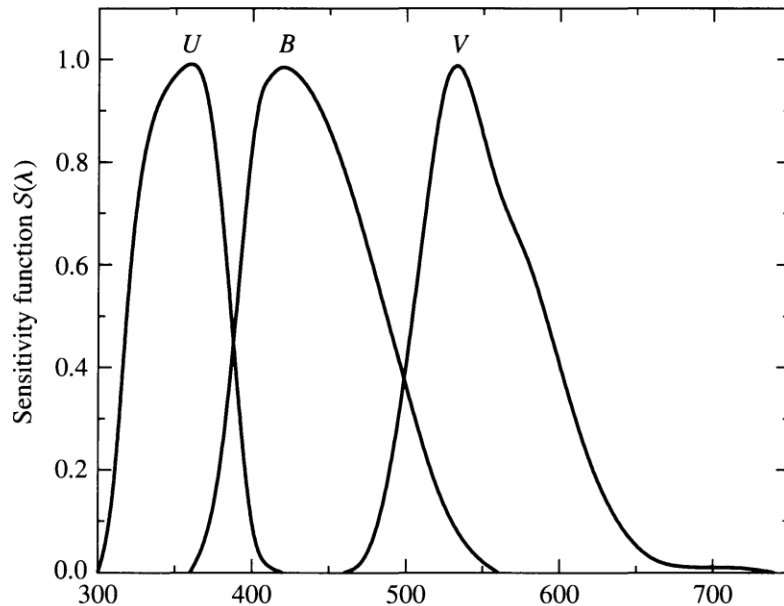
Solar Radiation Spectrum



Sun does behave like a very good blackbody.

Filters

Filters are classified by two parameters – the bandwidth, and the wavelength corresponding to center of the bandwidth.



U (Ultraviolet)

Bandwidth: 68 nm

λ_{cen} : 365 nm

B (Blue)

Bandwidth: 98 nm

λ_{cen} : 440 nm

V (Visual)

Bandwidth: 89 nm

λ_{cen} : 550 nm

Why Filters At All?

Why not collect all the light that we receive at the telescope?



Fine, we get the flux of the star. What next?



Find the distance, calculate the luminosity, and we are done!

Can we always find the distance?

What about those windows of atmospheric absorption?

There's a shorter route!

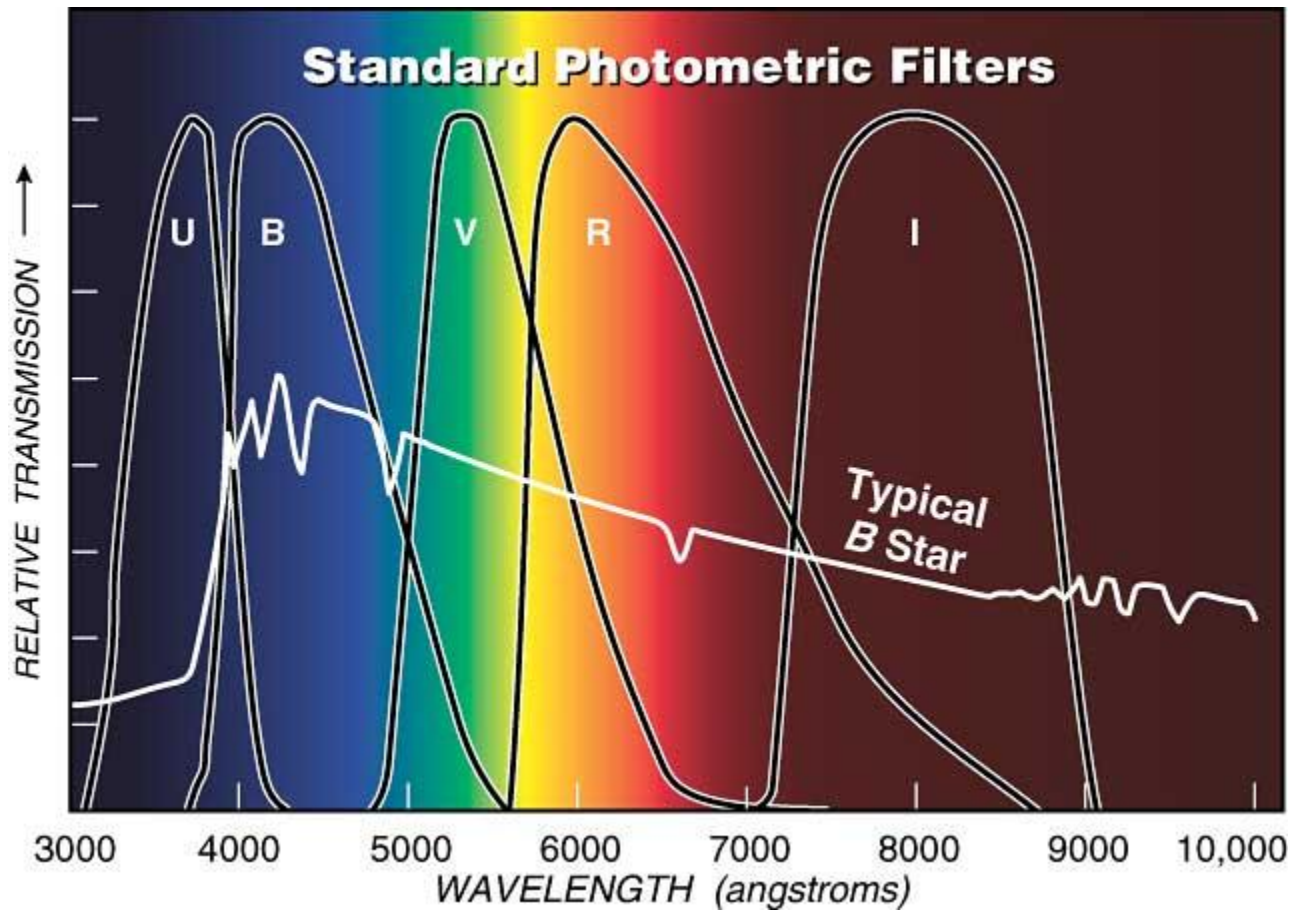
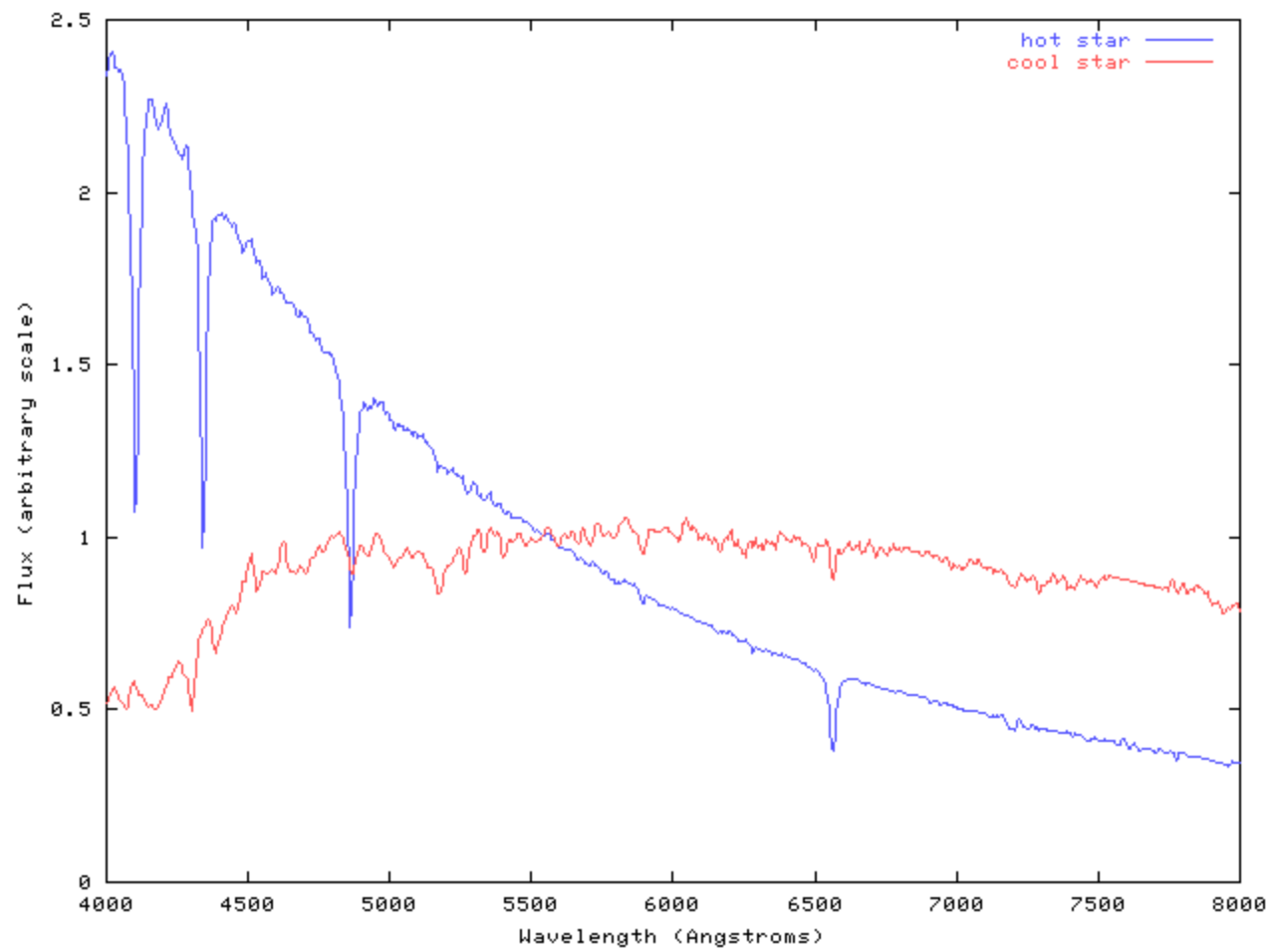
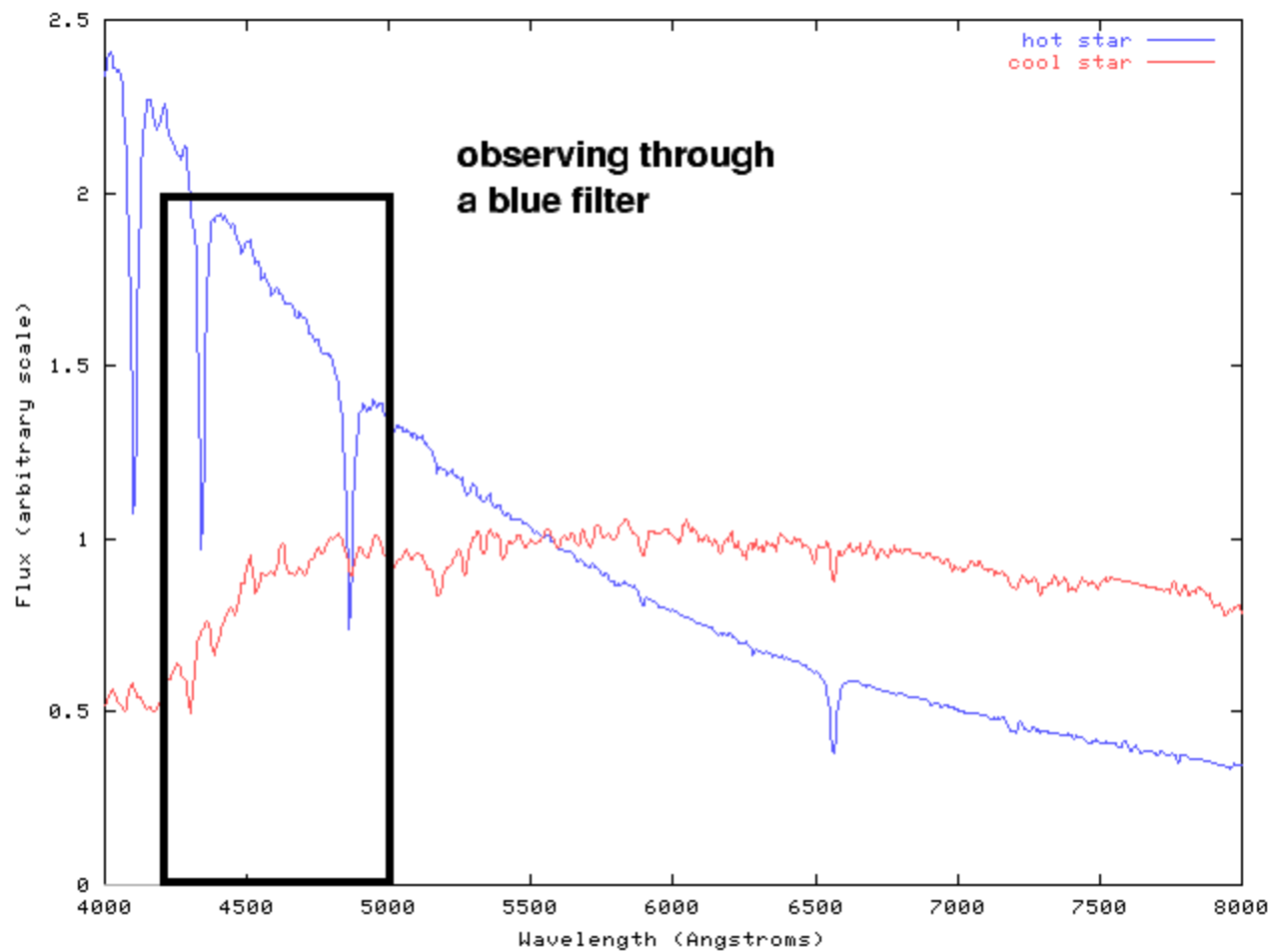


Image Source: Sky & Telescope





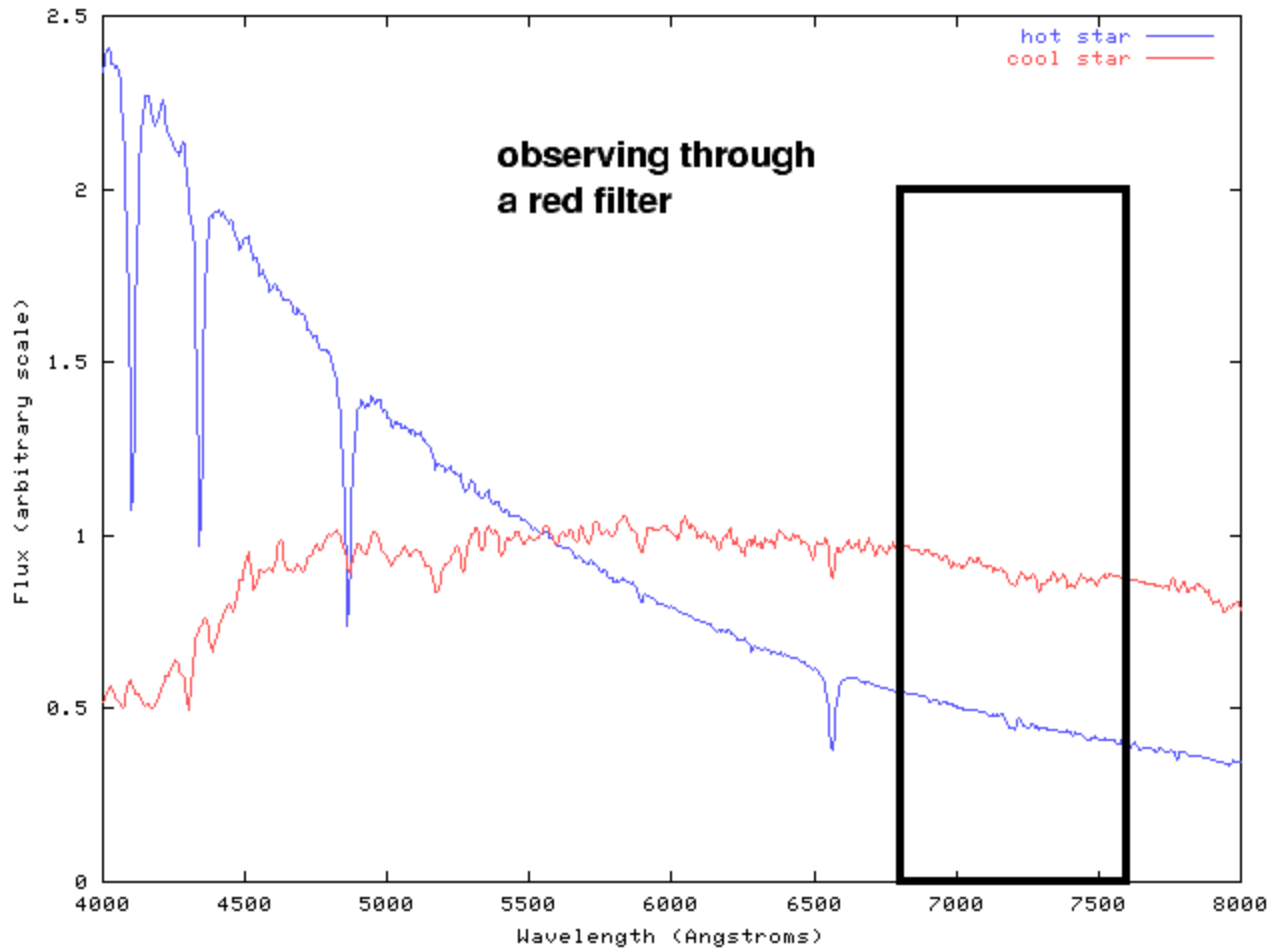


Figure Source: <http://spiff.rit.edu/classes/phys445/>

Color Indices

Color Index (or simply color) is the magnitude difference of a star in any two filters, for ex. U–B, or B–V.

$$U - B = M_U - M_B$$

$$B - V = M_B - M_V$$

Smaller the B – V color bluer the star.

The color index is independent of the distance of the star!

Apparent Magnitude and Flux

$$U = -2.5 \log_{10} \left(\int_0^{\infty} F_{\lambda} S_U d\lambda \right) + C_U$$

where C_U is a constant and S_U is a sensitivity function used to describe the fraction of the star Flux detected at that wavelength OR

$$U - B = -2.5 \log_{10} \left(\frac{\int F_{\lambda} S_U d\lambda}{\int F_{\lambda} S_B d\lambda} \right) + C_{U-B}$$

where $C_{U-B} = C_U - C_B$

Problem 3.19

Shaula is a bright blue-white subgiant star ($V = 1.62$) with a surface temperature of 22,000 K.

- (a) Use the values $C_{U-B} = -0.87$, and $C_{B-V} = 0.65$ and estimate the U-B and B-V color indices of Shaula. Compare your answers with the measured values, U-B = -0.90, and B-V = -0.23.
- (b) The parallax for this star is $0.00464''$. Determine the absolute magnitude of this star.

Ans (a) U-B = -1.08, B-V = -0.23
(b) -5.05

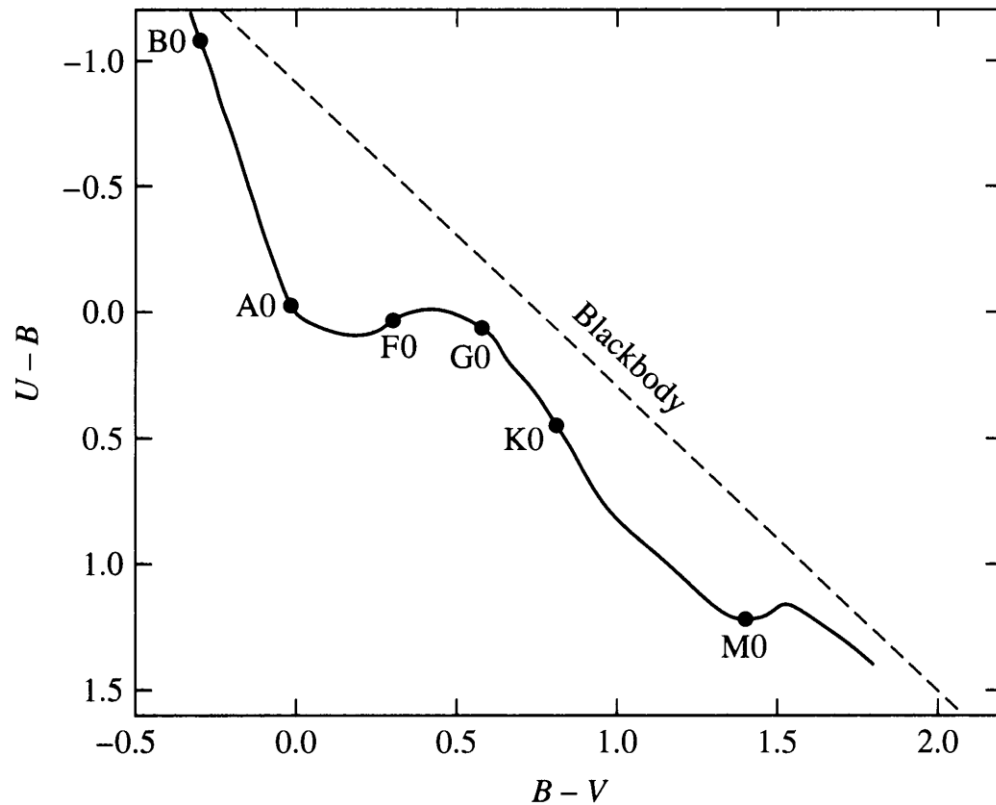
Bolometric Magnitudes & Bolometric Corrections

When apparent and absolute magnitudes are measured over all wavelengths, they are called bolometric magnitudes, and denoted by m_{bol} or M_{bol} .

Bolometric correction is the factor you add to a star's visual magnitude to get its bolometric magnitude.

$$\text{BC} = m_{\text{bol}} - V = M_{\text{bol}} - M_V$$

The Color-Color Diagram



Spectral Class Chart

Spectral Class	Intrinsic Color	Surface Temperature (K)
O	Blue	41,000
B	Blue	31,000
A	Blue-white	9,500
F	White	7,240
G	Yellow-white	5,920
K	Orange	5,300
M	Red	3,850

Assignment: 3.9, 3.15, 3.16, 3.17, 3.18, 3.19