126 GeV Higgs Boson Pair Production at the Linear Collider in the Noncommutative Space–Time

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We study the 126 GeV Higgs boson pair production through $e^+e^-$ collision in the noncommutative extension of the standard model using the Seiberg–Witten map of this to the first order of the noncommutative parameter $\Theta_{\mu\nu}$. The process is forbidden in the standard model at the tree level. We study the time-averaged cross-section of the pair production at TeV scale linear collider and investigate the sensitivity of the cross-section on the orientation angle $\eta$ and the noncommutative scale $\Lambda$. We found that $\Lambda$ lies in the range 0.5 TeV–1.0 TeV, which can be reached by the upcoming linear collider.

Keywords: Noncommutative space–time; standard model Higgs boson; $e^-e^+$ collider.

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1. Introduction

On 4 July 2012, the representatives of CMS and ATLAS groups of the large hadron collider at CERN have reported the observation of a scalar resonance of mass about 126 GeV at 4.9$\sigma$ and 5$\sigma$ confidence levels.$^{1,2}$ It matches with the properties of

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$^{a}$“The results are preliminary but the 4.9$\sigma$ signal at around 125 GeV were seen as dramatic. This is indeed a new particle. We know it must be a boson and it is the heaviest boson ever found,” said CMS experiment spokesperson Joe Incandela. “The implications are very significant and it is precisely for this reason that we must be extremely diligent in all of our studies and cross-checks.”
the standard model (SM) Higgs boson.\cite{3,6} The Higgs boson which gives masses to the charged fermions and the weak bosons $W^{\pm}, Z$ in the SM, has some unpleasant features associated with it. One of them is the hierarchy problem which arises from the instability of the Higgs mass between the two widely separated scales: the electroweak scale $M_{EW} (\sim \text{TeV})$ and the Planck scale $M_{Pl} (\sim 10^{16} \text{ TeV})$. A resolution to this problem requires the existence of beyond the SM physics which is possibly around or above the Higgs mass scale. It is therefore of supreme interest to see if the collider signals of the Higgs boson contain some imprints of new physics.

As a resolution of the Higgs hierarchy problem although theories such as supersymmetry, technicolor, unparticle physics which are phenomenologically appealing have been proposed, the idea of extra spatial dimensions with TeV scale gravity has drawn a lot of interest among the physics community.\cite{7} In a class of brane-world models\cite{8} where this TeV scale gravity is realized, one can principally expect to see some stringy effects and the signature of space–time noncommutativity in the TeV energy colliders such as large hadron collider (presently running) and linear collider (upcoming).

A lot of interests in the noncommutative (NC) field theories arose from the pioneering work by Snyder.\cite{9} It has drawn further attention recently due to developments connected to string theories in which the noncommutativity of space–time is an important characteristic of D-brane dynamics at low energy limit.\cite{10-12} Although Douglas et al.\cite{11} in their pioneering work have shown that noncommutative field theory is a well-defined quantum field theory, the question that remains is whether string theory predictions and the noncommutative effects can be seen at the energy scale attainable in present or near future experiments instead of the four-dimensional Planck scale $M_{Pl} \sim 10^{19} \text{ GeV}$. A notable work by Witten et al.\cite{13,14} suggests that one can see some stringy effects by lowering down the threshold value of commutativity to TeV, a scale which can be probed at the presently running LHC and at the upcoming collider such as linear collider (LC).

What is meant by noncommutative space–time? It means space and time no longer commute with each other, i.e. one cannot measure the space and time coordinates simultaneously with the same accuracy. Writing the space–time coordinates as operators we find

$$[\hat{X}_\mu, \hat{X}_\nu] = i\Theta_{\mu\nu},$$  \hspace{1cm} (1)

where the matrix $\Theta_{\mu\nu}$ is real and antisymmetric. The NC parameter $\Theta_{\mu\nu}$ has dimension of area and reflects the extent to which the space–time coordinates are noncommutative i.e. fuzzy. Furthermore, introducing a NC scale $\Lambda$, we rewrite Eq. (1) as

$$[\hat{X}_\mu, \hat{X}_\nu] = i\frac{\Theta_{\mu\nu}}{\Lambda^2},$$  \hspace{1cm} (2)

where $\Theta_{\mu\nu} (= c_{\mu\nu}/\Lambda)$ and $c_{\mu\nu}$ (antisymmetric in $\mu$ and $\nu$) has the same properties as $\Theta_{\mu\nu}$. To study an ordinary field theory in such a noncommutative space–time, one replaces all ordinary products among the field variables with Moyal–Weyl (MW)\cite{15}
This continues to exist in the noncommutative version of the charge quantization problem inherent in NC and others. Contrary to the Wilsonian renormalization, in the NC field theory the low energy theory does not get decoupled from the high energy dynamics: in the one-loop self-energy correction, the derivative (of field) coupling that arises with $\Theta^{\mu \nu}$ is restricted to $U(3) \otimes U(2) \otimes U(1)$ (see Chaichian et al.\textsuperscript{31}) and one requires a Higgs mechanism together with the introduction of additional gauge bosons in order to get the correct SM gauge group. In Ref. 32, Chaichian et al. discussed the problem of charge quantization in the NC gauge field theories and proposed a possible resolution of the same.\textsuperscript{9} A lot of phenomenological searches e.g. of the $Z \to \gamma \gamma$, $gg$ decays (processes which are forbidden in the SM and arising out from the triple neutral gauge boson interactions in the NCSM\textsuperscript{24–30}) have been reported.

\textsuperscript{9}We should mention that in Refs. 38, 41, 56 the charge quantization problem inherent in NC gauge field theories discussed and treated in Refs. 32 and 31, was dismissed by “mapping” three different noncommutative gauge field degrees of freedom to a single ordinary gauge field.\textsuperscript{33}

\[
(f \ast g)(z) = \exp \left( \frac{i}{2} \Theta^{\mu \nu} \partial_{\mu} \partial_{\nu} \right) f(x)g(y) \bigg|_{x=y=z}.
\]

Using this we can get the NCQED Lagrangian as

\[
\mathcal{L} = \frac{1}{2} i(\bar{\psi} \gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \gamma^\mu \psi) - m \bar{\psi} \gamma^\mu \psi - \frac{1}{4} F_{\mu \nu} \ast F^{\mu \nu},
\]

which are invariant under the following transformations:

\[
\psi(x, \Theta) \to \psi'(x, \Theta) = U \ast \psi(x, \Theta),
\]

\[
A_\mu(x, \Theta) \to A'_\mu(x, \Theta) = U \ast A_\mu(x, \Theta) \ast U^{-1},
\]

where $U = (e^{iA}) \ast$. In the NCQED Lagrangian (Eq. (4)) $D_\mu \bar{\psi} = \partial_\mu \bar{\psi} - i e A_\mu \ast \bar{\psi}$, $(D_\mu \bar{\psi}) = \partial_\mu \bar{\psi} + i e \bar{\psi} \ast A_\mu$, $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i e (A_\mu \ast A_\nu - A_\nu \ast A_\mu)$. In the MW approach the group closure property is only found to hold for the $U(N)$ gauge theories and the matter content is found to be in the (anti)fundamental and adjoint representations. An extensive work on noncommutative field theory including its renormalization exists in the literature.\textsuperscript{16–22} Contrary to the Wilsonian renormalization, in the NC field theory the low energy theory does not get decoupled from the high energy dynamics: in the one-loop self-energy correction, the derivative (of field) coupling that arises with $\Theta_{\mu \nu}$ in the interaction Lagrangian gives rise to mixing between the ultraviolet and the infrared limit, known as the UV/IR mixing problem.\textsuperscript{23,24} This continues to exist in the noncommutative version of QED, NCQED. In Ref. 23, it was shown that this UV/IR mixing is not so disastrous or it may not have any observable effects. Another interesting thing is the appearance of 3-photon and 4-photon vertices in NCQED which is analogous to the Yang–Mills theory. Hewett \textit{et al.}\textsuperscript{25–27} and others\textsuperscript{28,29} explored several processes, e.g. $e^+ e^- \to e^+ e^-$ (Bhabha), $e^- e^- \to e^- e^-$ (Möller), $e^- \gamma \to e^- \gamma$, $e^+ e^- \to \gamma \gamma$ (pair annihilation), $\gamma \gamma \to e^+ e^-$ and $\gamma \gamma \to \gamma \gamma$ in the context of NCQED. Conroy \textit{et al.}\textsuperscript{30} have investigated the process $e^+ e^- \to \mu^+ \mu^-$ in the context of NCQED and predicted a reach of $\Lambda = 1.7$ TeV. See also the work by Chaichian \textit{et al.} in this regard.

In an effort to construct the noncommutative standard model (NCSM) one is restricted to $U(3) \otimes U(2) \otimes U(1)$ (see Chaichian \textit{et al.}\textsuperscript{31}) and one requires a Higgs mechanism together with the introduction of additional gauge bosons in order to get the correct SM gauge group. In Ref. 32, Chaichian \textit{et al.} discussed the problem of charge quantization in the NC gauge field theories and proposed a possible resolution of the same.\textsuperscript{9} A lot of phenomenological searches e.g. of the $Z \to \gamma \gamma$, $gg$ decays (processes which are forbidden in the SM and arising out from the triple neutral gauge boson interactions in the NCSM\textsuperscript{24–30}) have been reported.
For an extensive discussion on the NC phenomenology in the Weyl–Moyal approach see Ref. 39.

There is a second approach which treats the space–time noncommutativity perturbatively via the Seiberg–Witten (SW) map expansion of the fields in terms of $\Theta$.\textsuperscript{11–15} The matter field $\psi$, the gauge field $A^\mu$ and the gauge transformation parameter $\Lambda_\alpha(x)$ in the noncommutative space–time can be expanded in terms of the commutative ones as a power series expansion in $\Theta$, i.e.

\begin{align}
\hat{\psi}(x, \Theta) &= \psi(x) + \Theta \psi^{(1)} + \Theta^2 \psi^{(2)} + \cdots, \\
\hat{A}_\mu(x, \Theta) &= A_\mu(x) + \Theta A_\mu^{(1)} + \Theta^2 A_\mu^{(2)} + \cdots, \\
\Lambda_\alpha(x, \Theta) &= \alpha(x) + \Theta \Lambda^{(1)}(x; \alpha) + \Theta^2 \Lambda^{(2)}(x; \alpha) + \cdots.
\end{align}

The advantage in the SW approach is that this construction can be applied to any gauge theory (including the standard model) in which matter can be in an arbitrary representation. We do not need to introduce any extra fields: particle content is the same as in the SM. Also note that the nontrivial phase factor (which arises after summing overall orders of $\Theta$ in the star product defined in the Weyl–Moyal plane), which gives rise to the UV/IR mixing, may not show up in the $\mathcal{N} = 4$ supersymmetric noncommutative gauge theories.\textsuperscript{40}

Using the SW technique, Calmet et al.\textsuperscript{41,42} first constructed a model with noncommutative gauge invariance which is known as the minimal noncommutative standard model (mNCSM). They listed the Feynman rules of the standard model interaction (modified) and new interactions which are absent in the SM. A lot of phenomenological searches, e.g. Bhabha and Möller scattering in the noncommutative space–time by Das et al.,\textsuperscript{43} neutrino–photon interaction in the noncommutative space–time and its impact on the cooling of stars,\textsuperscript{44} Quarkonia decay to two photons\textsuperscript{45} and $K \to \pi\gamma$ decay (processes forbidden by Lorentz invariance) in the noncommutative space–time,\textsuperscript{46} impact of noncommutative space–time on the primordial nucleosynthesis and ultrahigh-energy cosmic ray\textsuperscript{47,48} have been made and are available in the literature.

The noncommutativity parameter $\Theta_{\mu\nu}$ may be an elementary constant in nature that has a fixed direction in a specific coordinate system fixed to the celestial sphere. The laboratory frame which is located on the earth is moving by earth’s rotation. So we should take into account the apparent time variation of $\Theta_{\mu\nu}$ in the laboratory frame when we make any phenomenological investigation of scattering or decay of particles on the surface of the earth. The effect of earth’s rotation on noncommutative phenomenology were considered in several earlier studies.\textsuperscript{39–52} Here we will investigate the 126 GeV Higgs pair production in the noncommutative space–time taking the earth’s rotation into account. In addition if space–time is anisotropic due to the noncommutativity, then a probe to the magnitude of the length scale (scale of anisotropy) and the specific direction of $\Theta_{\mu\nu}$ may be very interesting both from an experimental and theoretical point of view. We may determine the direction of $\Theta_{\mu\nu}$ by studying the behavior of several time averaged observables.

\begin{align}
\hat{\psi}(x, \Theta) &= \psi(x) + \Theta \psi^{(1)} + \Theta^2 \psi^{(2)} + \cdots, \\
\hat{A}_\mu(x, \Theta) &= A_\mu(x) + \Theta A_\mu^{(1)} + \Theta^2 A_\mu^{(2)} + \cdots, \\
\Lambda_\alpha(x, \Theta) &= \alpha(x) + \Theta \Lambda^{(1)}(x; \alpha) + \Theta^2 \Lambda^{(2)}(x; \alpha) + \cdots.
\end{align}
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\[ e^+ (p_1) - e^- (p_2) + H(p_3) = e^+ (p_1) - e^- (p_2) + H(p_3) + \gamma H(p_4) + e^+ (p_1) - e^- (p_2) + H(p_3) + Z H(p_4) \]

Fig. 1. Feynman diagrams for \( e^+ e^- \rightarrow \gamma, Z \rightarrow HH \) in the NCSM.

In Sec. 2, we present the cross-section of \( e^+ e^- \rightarrow \gamma, Z \rightarrow HH \) in the NCSM. In Sec. 3, we make a detailed numerical analysis of the pair production cross-section (time-averaged) and its sensitivities on the noncommutative parameters. We discuss the prospects of TeV scale noncommutative geometry. Finally, we conclude in Sec. 4.

2. Higgs Pair Production at the Future Linear Collider

Since there is no direct coupling of a photon (\( \gamma \)) or a \( Z \) boson with a pair of Higgs boson, the pair production of Higgs boson through \( e^+ - e^- \) annihilation is forbidden in the SM at the tree level. So an excess in the predicted event rate may be interpreted as a signature of new physics. Supersymmetry and extra dimensional models are front runners (see Ref. 53 and references therein). In the nonminimal NCSM (nmNCSM) scenario, we explore the potential feasibility of the channel \( e^+ e^- \rightarrow \gamma, Z \rightarrow HH \) for possible Higgs pair production. In an earlier work, \(^5\) we have investigated the Higgs pair production in noncommutative space–time without considering the effect of earth rotation. Here we include the effect of earth’s rotation.

The Feynman diagrams for the process \( e^+ e^- \rightarrow \gamma, Z \rightarrow HH \) are shown in Fig. 1.

The scattering amplitude for the above process using \( O(\Theta) \) Feynman rules (see App. A for details) can be written as

\[ A = A_\gamma + A_Z, \]

where \( A_\gamma \) and \( A_Z \) can be calculated as

\[ A_\gamma = \frac{-i\pi\alpha m_H^2}{s} \left[ \bar{v}(p_2)\gamma_\mu u(p_1) \right] \times (k\Theta)^\mu \left[ 1 + \frac{i}{2}(p_2\Theta p_1) \right], \]

\[ A_Z = \frac{-i\pi\alpha m_Z^2}{\sin^2(2\theta_W)s_Z} \left[ \bar{v}(p_2)\gamma_\mu (4\sin^2(\theta_W) - 1 + \gamma^5) u(p_1) \right] \times (k\Theta)^\mu \left[ 1 + \frac{i}{2}(p_2\Theta p_1) \right]. \]

Here \( s = k^2 \) with \( k = p_1 + p_2 = p_3 + p_4 \) and \( s_Z = s - m_Z^2 + i\Gamma_Z m_Z \). \( \alpha = e^2/4\pi \) and \( \theta_W \) is the Weinberg angle. \( m_H \) is the Higgs mass, \( m_Z \) and \( \Gamma_Z \) are the mass and...
decay width of the $Z$ boson. Using Eqs. (11) and (12), we find the spin averaged squared-amplitude as

$$|A|^2 = |A_\gamma|^2 + |A_Z|^2 + 2 \text{Re}(A_{\gamma}^* A_Z) = \frac{1}{4} \sum_{\text{spin}} |A|^2,$$

(13)

where several terms of Eq. (13) are given in App. C.

The noncommutative parameter $\Theta_{\mu\nu}$ is considered to be a fundamental constant in nature. Its direction is fixed with respect to an inertial (nonrotating) coordinate system (which can be a celestial coordinate system). Now our experiment is done in the laboratory coordinate system which is located on the surface of the earth and is moving by the earth’s rotation. This gives rise to an apparent time variation of the components of $\Theta_{\mu\nu}$ which should be taken into account while making any phenomenological investigation.

Here we will follow Kamoshita’s work$^{50}$ for the notations. Let $\hat{i}_X$, $\hat{j}_Y$, and $\hat{k}_Z$ be the orthonormal bases of the primary (nonrotating) coordinate system $(X–Y–Z)$. Then in the laboratory coordinate system $(\hat{i}–\hat{j}–\hat{k})$, the bases vectors of the primary (nonrotating) coordinate system can be written as

$$\hat{i}_X = \begin{pmatrix} c_\delta s_\zeta + s_\delta s_\eta c_\zeta \\ c_\eta c_\delta + s_\delta s_\eta s_\zeta \\ s_\eta s_\delta - s_\delta c_\eta c_\zeta \end{pmatrix}, \quad \hat{j}_Y = \begin{pmatrix} -c_\eta c_\zeta + s_\delta s_\eta s_\zeta \\ s_\eta c_\delta + s_\delta s_\eta s_\zeta \\ c_\eta s_\delta - s_\delta c_\eta c_\zeta \end{pmatrix}, \quad \hat{k}_Z = \begin{pmatrix} -c_\delta s_\eta \\ s_\delta \\ c_\delta c_\eta \end{pmatrix}.$$

Here we have used the abbreviations $c_\delta = \cos \delta$, $s_\delta = \sin \delta$, etc. In Fig. 2, the primary $(X–Y–Z)$ and the laboratory $(\hat{i}–\hat{j}–\hat{k})$ coordinate system are shown. Note that the primary $Z$-axis is along the axis of earth’s rotation and $(\delta, a)$ defines the
location of $e^- - e^+$ experiment on the earth, with $-\pi/2 \leq \delta \leq \pi/2$ and $0 \leq a \leq 2\pi$. Because of earth’s rotation the angle $\zeta$ (see Fig. 2) increases with time and the detector comes to its original position after a cycle of one complete day, one can define $\zeta = \omega t$ with $\omega = 2\pi/T_{\text{day}}$ and $T_{\text{day}} = 23\,\text{h}\,56\,\text{m}\,4.09053\,\text{s}$.

Using all these, the electric and the magnetic components of the NC parameter $\Theta_{\mu\nu}$ in the primary system is given by

$$
\Theta_E = \Theta_E \sin \eta_E \cos \xi_E \hat{i}_X + \Theta_E \sin \eta_E \sin \xi_E \hat{j}_Y + \Theta_E \cos \eta_E \hat{k}_Z ,
$$

$$
\Theta_B = \Theta_B \sin \eta_B \cos \xi_B \hat{i}_X + \Theta_B \sin \eta_B \sin \xi_B \hat{j}_Y + \Theta_B \cos \eta_B \hat{k}_Z ,
$$

with

$$
\Theta_E = (\Theta_0^1, \Theta_0^2, \Theta_0^3) , \quad \Theta_B = (\Theta_2^3, \Theta_3^1, \Theta_1^2)
$$

and

$$
\Theta_E = |\Theta_E| = \frac{1}{\Lambda_E} , \quad \Theta_B = |\Theta_B| = \frac{1}{\Lambda_B} .
$$

Here $(\eta, \xi)$ specifies the direction of the NC parameter $\Theta_{\mu\nu}$ with respect to the primary coordinate system with $0 \leq \eta \leq \pi$ and $0 \leq \xi \leq 2\pi$. In above $\Theta_E$ and $\Theta_B$ are the model parameters and the energy scales defined by $\Lambda_E = 1/\sqrt{\Theta_E}$ and $\Lambda_B = 1/\sqrt{\Theta_B}$ can be probed for different processes.

The spin-averaged squared-amplitude of the $e^+ e^- \to ZZ$ scattering is given by

$$
|A|^2 = |A_{\gamma}|^2 + |A_Z|^2 + 2 \text{Re}(A_{\gamma} A_Z^*) .
$$

The direct and interference terms in Eq. (18) are given in App. C. Since it is difficult to get the time-dependent data, we take the average of the cross-section or its distribution over the sidereal day $T_{\text{day}}$. We introduce the time averaged observables as follows:

$$
\left\langle \frac{d^2\sigma}{d\cos \theta \, d\phi} \right\rangle_T = \frac{1}{T_{\text{day}}} \int_0^{T_{\text{day}}} \frac{d\sigma}{d\cos \theta \, d\phi} dt ,
$$

$$
\left\langle \frac{d\sigma}{d\cos \theta} \right\rangle_T = \frac{1}{T_{\text{day}}} \int_0^{T_{\text{day}}} \frac{d\sigma}{d\cos \theta} dt ,
$$

$$
\left\langle \frac{d\sigma}{d\phi} \right\rangle_T = \frac{1}{T_{\text{day}}} \int_0^{T_{\text{day}}} \frac{d\sigma}{d\phi} dt ,
$$

$$
\langle \sigma \rangle_T = \frac{1}{T_{\text{day}}} \int_0^{T_{\text{day}}} \sigma \, dt .
$$
where
\[
\sigma = \int_{-1}^{1} d(\cos \theta) \int_{0}^{2\pi} d\phi \frac{d\sigma}{d\cos \theta d\phi},
\]
\[
\frac{d\sigma}{d\cos \theta} = \int_{0}^{2\pi} d\phi \frac{d\sigma}{d\cos \theta d\phi},
\]
\[
\frac{d\sigma}{d\phi} = \int_{-1}^{1} d(\cos \theta) \frac{d\sigma}{d\cos \theta d\phi}.
\]

In the previous expressions,
\[
\frac{d^2\sigma}{d\cos \theta d\phi} = \frac{1}{64\pi^2 s} \left(1 - \frac{4m_H^2}{s}\right)^{1/2} |A|^2,
\]
where \(\sigma = \sigma(\sqrt{s}, \Lambda, \theta, \phi, t)\). The time dependence in the cross-section or its distribution enters through the NC parameter \(\Theta(= \Theta_E)\) which changes with the change in \(\zeta = \omega t\). The angle parameter \(\xi\) appears in the expression of \(\Theta\) through \(\cos(\omega t - \xi)\) or \(\sin(\omega t - \xi)\) (Ref. 50) as the initial phase for time evolution disappears in the time averaged observables. So one can deduce \(\Theta_E\), i.e. \(\Lambda_E\) and the angle \(\eta_E\) from the time-averaged observables.

3. Numerical Analysis

Before making a detailed analysis, let us make some general remarks regarding the observation of noncommutative effects. Since we assume \(c_{\mu\nu} = (c_{0i}, c_{ij}) = (\xi_i, \epsilon_{ijk} \chi^k)\), where \(\xi_i = (E_i)\) and \(\chi_k = (B)_k\) are constant vectors in a frame that is stationary with respect to fixed stars, the vectors \(E\) and \(B\) point in fixed directions which are the same in all frames of reference. However, as the earth rotates around its axis and revolves around the Sun, the direction of \(E\) and \(B\) will change continuously with time dependence which is a function of the coordinates of the laboratory. The observables that are measured will thus show a characteristic time dependence. It is important to be able to measure this time dependence to verify such noncommutative theories. In one of our earlier work on pair production of Higgs boson, we have assumed the vectors \(E = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})\) and \(B = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})\), i.e. they behave like constant vectors,\(^{54}\) which happens to be the case at some instant of time. In the present work, we analyze in detail the effect of earth’s rotation. We probe the characteristic NC scale \(\Lambda\) and the orientation angle \(\eta\) (\(= \eta_E\)) using the time-averaged observables defined in the laboratory coordinate system. We set the laboratory coordinate system by taking \((\delta, a) = (\pi/4, \pi/4)\), which is the OPAL experiment at LEP.

3.1. Pair production cross-section in the NCSM

As mentioned earlier, because of the absence of any tree level couplings with photon or \(Z\) boson with a pair of Higgs boson, the Higgs pair production is highly suppressed. Any sizeable number of such events if observed at the linear collider, may
point towards the existence of new physics and the nmNCSM is a plausible candidate for new physics. In this section we make a detailed numerical study of the time-averaged cross-section and its dependence on the NC scale $\Lambda$ and the orientation angle $\eta$ (the orientation angle of the NC (electric) vector with the axis of earth’s rotation) which are shown in several plots.

In Fig. 3, we have plotted the time-averaged cross-section $\langle \sigma \rangle_T$ as a function of the orientation angle $\eta$ of the NC vector $\Theta_E$ for different machine energy. We have taken the Higgs mass $m_H = 126$ GeV. In the left (right) panel the lowermost, next to it and the topmost curve corresponds to $\Lambda = 1000, 700$ and 500 GeV with the machine energy fixed at $E_{com} = 500 (1000)$ GeV. From the plots we see that $\langle \sigma \rangle_T$ is larger for smaller $\Lambda$ value. It is maximum at $\eta = 0$ and $\pi$ and is minimum at $\eta = \pi/2$. This shows the strong dependence of the pair production on the orientation angle $\eta$ of the NC vector.

In Fig. 4, we have shown the total cross-section $\langle \sigma \rangle_T$ as a function of the NC scale $\Lambda$. The machine energy is fixed at $E_{com} = 500 (1000)$ GeV in the left (right) panel, respectively. In each panel the topmost curve, next to it and the lowermost
Table 1. The yearly number of events (the NC signals) with Λ (in GeV) are shown. The integrated luminosity of the LC is assumed to be \( L = 500 \text{ fb}^{-1} \). The orientation angle \( \eta \) of the NC vector is chosen to be 0(\( \pi \)). The machine energy \( E_{\text{com}} \) is expressed in GeV.

<table>
<thead>
<tr>
<th>( E_{\text{com}} ) (GeV)</th>
<th>( \Lambda ) (GeV)</th>
<th>( \sigma ) (fb)</th>
<th>( \mathcal{L} ) (fb(^{-1}))</th>
<th>( N ) (yr(^{-1}))</th>
<th>( E_{\text{com}} ) (GeV)</th>
<th>( \Lambda ) (GeV)</th>
<th>( \sigma ) (fb)</th>
<th>( \mathcal{L} ) (fb(^{-1}))</th>
<th>( N ) (yr(^{-1}))</th>
</tr>
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</tbody>
</table>

Fig. 5. \( \langle \sigma \rangle_T \) (fb) as a function of \( E_{\text{com}} \) (GeV) is shown. In the left panel we set \( \eta = 0 \) and the three curves correspond to \( \Lambda = 500, 700 \) and 1000 GeV, respectively. In the right panel the three curves correspond to \( \eta = 0, \pi/4 \) and \( \pi/2 \), respectively with \( \Lambda = 500 \) GeV. We set the Higgs mass at \( m_H = 126 \) GeV for both plots.

curve corresponds to \( \eta = 0 \) (or \( \pi \)), \( \pi/4 \) and \( \pi/2 \), respectively. For a given \( \eta \) the cross-section decreases with the increase in \( \Lambda \) and for a given \( \Lambda \) the cross-section is maximum for \( \eta = 0 \) (or \( \pi \)) and decreases otherwise (see Fig. 4). Assuming the LC luminosity (integrated) \( \mathcal{L} = 500 \text{ fb}^{-1} \), we predict the number of events of Higgs pair production in the NCSM which are shown in Table 1. In Table 1, we have shown the number of events as a function of \( \Lambda \) corresponding to the machine energy \( E_{\text{com}} = \sqrt{s} \) = 500 GeV and 1000 GeV, respectively.

We see that as \( \Lambda \) increases from 500–1000 GeV, the number of events \( N \) (yr\(^{-1}\)) of Higgs boson pair production decreases from 41 (11) per year to 3 (1) per year for the machine energy \( E_{\text{com}} = 500 \) (1000) GeV. So a maximum of 41 and 11 events (NC signal) per year are expected to be observed at the upcoming linear collider corresponding to the machine energy \( E_{\text{com}} = 500 \) GeV and 1000 GeV with an integrated luminosity \( \mathcal{L} = 500 \text{ fb}^{-1} \).

3.2. Total cross-section as a function of the machine energy in the NCSM

In Fig. 5, we have shown the time-averaged cross-section \( \langle \sigma \rangle_T \) as a function of the machine energy \( E_{\text{com}} \). We have set the Higgs mass at \( m_H = 126 \) GeV for both plots. In the left panel we set \( \eta = 0 \) and the three different curves, respectively stands for \( \Lambda = 500, 700 \) and 1000 GeV. In the right panel we have shown three
Fig. 6. (Color online) The variation of the cross-section \( \sigma(e^- e^+ \gamma \rightarrow HH) \) (fb) is shown with time \( t \) over a complete day \( T_{\text{day}} \). For the left (right) panel the machine energy is fixed at \( E_{\text{com}} = 500 \) (1000) GeV and the Higgs mass at \( m_H = 126 \) GeV. We set the NC scale at \( \Lambda = 500 \) GeV and the angle \( \eta \) is taken as 0, \( \pi/4 \), \( \pi/2 \) and \( 3\pi/4 \), respectively.

3.3. Time varying total cross-section

Next we look at the time-dependent behavior of the Higgs pair production cross-section. In Fig. 6, we have shown the variation of the cross-section with time over a complete day \( T_{\text{day}} \) corresponding to \( \eta = 0 \) (horizontal red line), \( \pi/4 \) (blue curve), \( \pi/2 \) (black curve) and \( 3\pi/4 \) (brown curve) and the NC scale \( \Lambda = 500 \) GeV. In the left (right) panel we have set the machine energy at \( E_{\text{com}} = 500 \) (1000) GeV and the Higgs mass at \( m_H = 126 \) GeV. Interestingly, the production cross-section have several maxima and minima during different times of the day. The pattern of each plot strongly depends on the value of \( \eta \).

3.4. Lower bound on the NC scale \( \Lambda \)

Assuming the machine luminosity \( L = 500 \) fb\(^{-1} \) and the orientation angle of the NC vector \( \eta = 0 \) or \( \pi \), we consider the following two cases:

- Case I: The machine energy is fixed at \( E_{\text{com}} = 500 \) GeV. The contour plots are obtained corresponding to \( N = 40 \) (yr\(^{-1} \)) and \( N = 5 \) (yr\(^{-1} \)).
- Case II: The machine energy is fixed at \( E_{\text{com}} = 1000 \) GeV. Two contour plots corresponding to \( N = 12 \) (yr\(^{-1} \)) and \( N = 1 \) (yr\(^{-1} \)) are obtained.

In Fig. 7, we have shown the contour plots. The left and right panel corresponds to Case I and Case II, respectively. From the point of intersection of the horizontal line (the 126 GeV mass Higgs line) with the contour curve for a given \( N \), we obtain the following lower bound on \( \Lambda \):

- In Case I, we find the lower bound on \( \Lambda = 498 \) GeV and 955 GeV corresponding to \( N = 41 \) yr\(^{-1} \) and 3 yr\(^{-1} \).
- In Case II we find the lower bound on \( \Lambda = 500 \) GeV and 900 GeV corresponding to \( N = 11 \) yr\(^{-1} \) and 1 yr\(^{-1} \).
4. Conclusion

We have studied the pair production of 126 GeV Higgs boson in the background of noncommutative space–time using the effect of earth rotation. Working within the nmNCSM, we found that the time-averaged cross-section of the pair production strongly depends on the orientation angle $\eta$: it is maximum at $\eta = 0$, $\pi$ and is minimum at $\eta = \pi / 2$. The cross-section $\langle \sigma \rangle_T$ and the number of events $N (\text{yr}^{-1})$ decreases with the increase in $\Lambda$. For example corresponding to $E_{\text{com}} = 500$ GeV and taking the integrated luminosity of the linear collider $L = 500 \text{ fb}^{-1}$, we see $N$ drops from 41 $\text{yr}^{-1}$ to 3 $\text{yr}^{-1}$ as $\Lambda$ falls from 500 GeV to 1000 GeV which can be compared with almost no events in the case of SM. We also plot the time-dependent cross-section $\sigma$ as a function of time $t$ over a complete day $T_{\text{day}}$ corresponding to different $\eta$ at a particular machine energy and find that it changes quite significantly with times in a day depending on the value of $\eta$. Finally, we obtain the contour plots in the plane of $m_H - \Lambda$ corresponding to the event rate $N (\text{yr}^{-1}) = 41$ (11) and 3 (1) with $E_{\text{com}} = 500$ (1000) GeV and find the lower bound $\Lambda = 498$ (500) GeV and $\Lambda = 955$ (900) GeV, respectively.

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Appendix A. Feynman Rules to Order $\mathcal{O}(\Theta)$

The Feynman rule for the $f(p_{\text{in}}) - f(p_{\text{out}}) - \gamma(k)$ vertex is

$$ieQ_f\, \gamma_{\mu} + \frac{1}{2}eQ_f\left[(p_{\text{out}}\Theta p_{\text{in}})\gamma_{\mu} - (p_{\text{out}}\Theta)_{\mu}(p_{\text{in}} - m_f) - (p_{\text{out}} - m_f)(\Theta p_{\text{in}})_{\mu}\right]$$

and for the $f(p_{\text{in}}) - f(p_{\text{out}}) - Z(k)$ vertex is

$$e\sin 2\theta_W\left[i\gamma_{\mu} \Gamma_{\pm} + e\sin 2\theta_W \times \left[(p_{\text{out}}\Theta p_{\text{in}})\gamma_{\mu} - (p_{\text{out}}\Theta)_{\mu}(p_{\text{in}} - m_f) - (p_{\text{out}} - m_f)\Gamma_{\pm} - (p_{\text{out}} - m_f)(\Theta p_{\text{in}})_{\mu}\right]\right].$$

Here $\Gamma_{\pm} = (e\gamma_\pm + eA\gamma_5)$ and $p_{\text{out}}(\Theta p_{\text{in}}) = p_{\text{out}}\Theta_{\mu\nu}p_{\text{in}}^{\nu} = -p_{\text{in}}\Theta p_{\text{out}}$. At the vertex the momentum conservation reads as $p_{\text{in}} + k = p_{\text{out}}$. Similarly, the Feynman rule for the interaction vertex $H(p_3) - H(p_4) - Z(k)$ is

$$\frac{g m_H^2(k\Theta)_{\mu}}{4 \cos \theta_W}$$

and for the vertex $H(p_3) - H(p_4) - \gamma(k)$ is

$$\frac{e m_H^2(k\Theta)_{\mu}}{4}.$$

In the above expressions, $(k\Theta)_{\mu} = k^\nu\Theta_{\nu\mu}$.

Appendix B. Momentum Prescriptions and Dot Products

We work in the center of momentum frame where the four momenta of the incoming and outgoing particles are given by

$$p_1 = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right),$$

$$p_2 = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right),$$

$$p_3 = \left(\frac{\sqrt{s}}{2}, k', \sin \theta \cos \phi, k' \sin \theta \sin \phi, k' \cos \theta\right),$$

$$p_4 = \left(\frac{\sqrt{s}}{2}, -k', \sin \theta \cos \phi, -k' \sin \theta \sin \phi, -k' \cos \theta\right),$$

$$k' = \frac{\sqrt{s}}{2} \sqrt{1 - \frac{4m_H^2}{s}},$$

where $\theta$ is the scattering angle made by the three-momentum vector $p_3$ of $H(p_3)$ with the $+ve\hat{z} (= \hat{k})$ axis (the three-momentum direction of the incoming electron) and $\phi$ is the azimuthal angle.

The antisymmetric NC tensor $\Theta_{\mu\nu} = (\Theta_E, \Theta_B)$, i.e. it has three electric and three magnetic components. The $s$-channel driven muon pair production in electron–positron collision is found to be sensitive only to the $\Theta_E$ vector and hence one obtain constraints on $\Lambda_E (= \Lambda, \text{say})$. In the laboratory frame (with $\eta = \eta_E$, $\Lambda_E$ =
\( \xi = \xi_E \), the electric NC vector \( \Theta_E \) can be written as
\[
\Theta_E = \Theta_E \sin \eta \cos \xi \hat{i}_X + \Theta_E \sin \eta \sin \xi \hat{j}_Y + \Theta_E \cos \eta \hat{k}_Z
\]
\[= \Theta_{Ez}^\text{lab} \hat{i} + \Theta_{Ey}^\text{lab} \hat{j} + \Theta_{Ez}^\text{lab} \hat{k}, \tag{B.6} \]
where
\[
\Theta_{Ez}^\text{lab} = \Theta_E \left( s_\eta c_\xi (c_\delta s_\zeta + s_\xi s_\delta c_\zeta) + s_\eta s_\xi (-c_\delta c_\zeta + s_\xi s_\delta) - c_\eta c_\delta s_\zeta \right),
\]
\[
\Theta_{Ey}^\text{lab} = \Theta_E \left( s_\eta c_\delta c_\zeta + s_\xi s_\delta + c_\eta s_\zeta \right),
\]
\[
\Theta_{Ez}^\text{lab} = \Theta_E \left( s_\eta c_\xi (s_\delta s_\zeta - s_\delta c_\zeta) - s_\eta s_\xi (s_\delta c_\zeta + s_\delta s_\zeta) + c_\eta c_\delta c_\zeta \right),
\]
with (see the main section)
\[
\Theta_E = |\Theta_E| = \frac{1}{X^2}. \tag{B.8} \]
In above we have used abbreviations \( s_\eta = \sin \eta, c_\xi = \cos \xi \), etc. As mentioned earlier, \( (\eta, \xi) \) specifies the direction of \( \Theta_E \) with respect to the primary coordinate system with \( 0 \leq \eta \leq \pi \) and \( 0 \leq \xi \leq 2\pi \). Using these we find
\[
p_2 \Theta_{p_1} = -\frac{s_\xi}{2} \Theta_{Ez}^\text{lab}, \tag{B.9}
\]
\[
(k\Theta)_0 = 0, \tag{B.10}
\]
\[
(k\Theta)_1 = -\sqrt{s} \Theta_{Ey}^\text{lab}, \tag{B.11}
\]
\[
(k\Theta)_2 = -\sqrt{s} \Theta_{Ey}^\text{lab}, \tag{B.12}
\]
\[
(k\Theta)_3 = -\sqrt{s} \Theta_{Ez}^\text{lab}, \tag{B.13}
\]
where \( \Theta_{Ez}^\text{lab}, \Theta_{Ey}^\text{lab} \) and \( \Theta_{Ez}^\text{lab} \) are defined above. Noting \( k = p_1 + p_2 = p_3 + p_4 \), we find \( p_1 \cdot p_2 = s/2 = p_3 \cdot p_4 \) and \( (k\Theta) \cdot (k\Theta) = -s (\Theta_{Ez}^\text{lab} \cdot \Theta_{Ez}^\text{lab}) \).

**Appendix C. Spin-Averaged Squared Amplitude**

The various components of Eq. (13) are found to be
\[
|A_3|^2 = -\frac{\pi^2 \alpha^2 m_H^4}{8} \mathcal{F}, \tag{C.1}
\]
\[
|A_2|^2 = \frac{\pi^2 \alpha^2 m_H^4}{\sin^4(2\theta_W)} \left[ 1 + (4\sin^2 \theta_W - 1)^2 \right] \mathcal{F}, \tag{C.2}
\]
\[
2 \text{Re}(A_1^\dagger A_2) = \frac{2\pi^2 \alpha^2 m_H^4}{\sin^4(2\theta_W)} \frac{(4\sin^2 \theta_W - 1)(s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2} \mathcal{F}. \tag{C.3}
\]
The overall factor \( \mathcal{F} \) is given by
\[
\mathcal{F} = \left[ 2(p_1 \cdot p_2)^2 + (p_1 \cdot p_2)((k\Theta) \cdot (k\Theta)) \right]. \tag{C.4}
\]
The dot product terms appearing above are listed in App. B.
References

2. CMS Collab., CMS-PAS-HIG-12-008.
P. K. Das & A. Prakash