INVENTORY SYSTEMS

INVENTORY

- Stock of good items maintained to full fill present & future need of an organization

Manufacturer:- Raw material stock, spare parts, semi furnished goods

Hospital :- Stock of drugs

Book-Store :- Books & Stationary

Why inventory is maintained?

- It provides service to the customer immediately or at a short period
- Due to absence of stock the company may have to pay high price because of piecewise purchasing
- Maintaining of inventory may earn price discount because of bulk purchasing
- Inventory acts as a buffer stock when raw material are received late so many sale orders are likely to be rejected

Various costs involved

- **Procurement Cost:-**

  Associated costs are

  a) Set up cost:- K per cycle

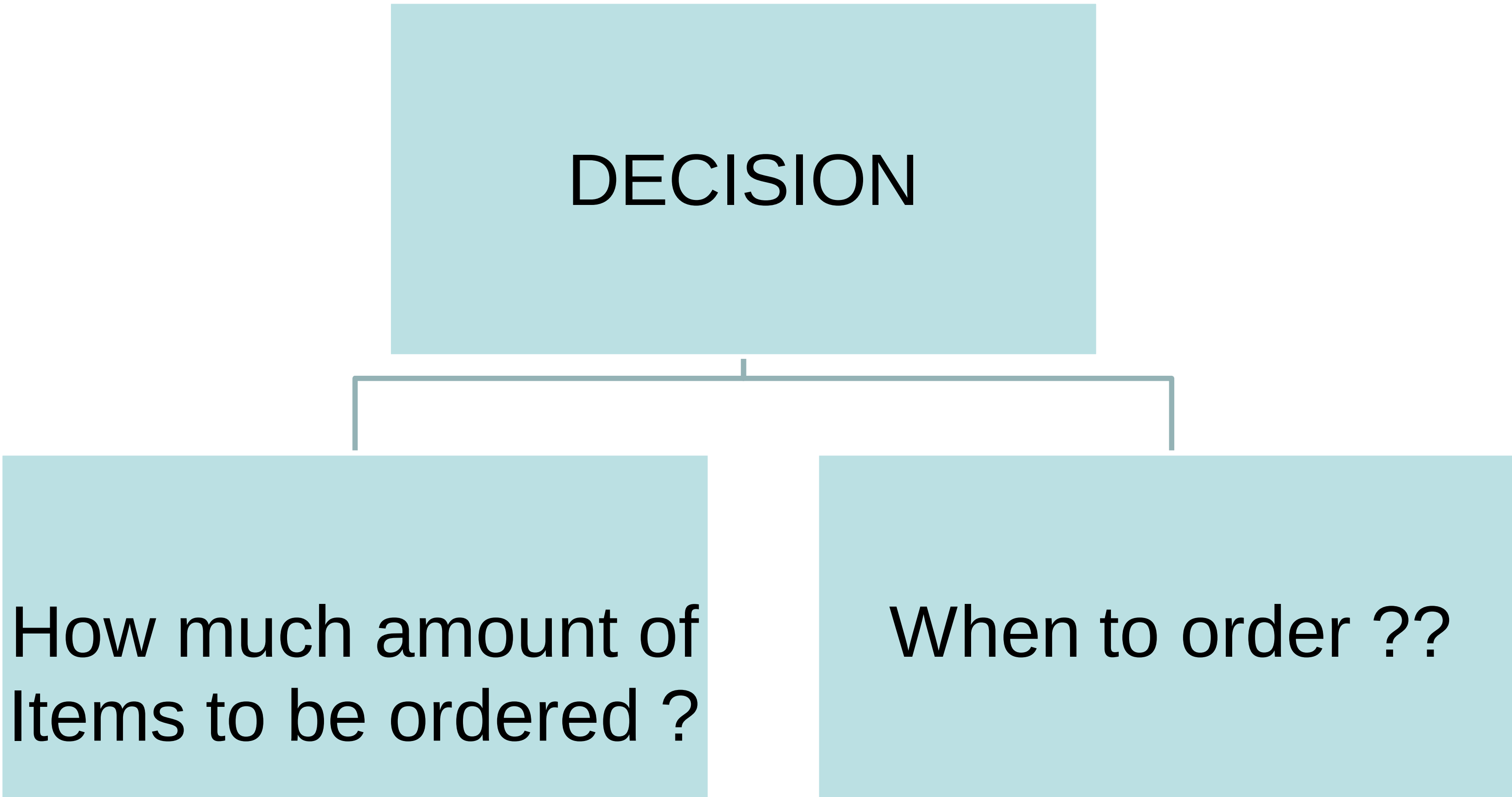
  b) Purchase or production cost:- Co per unit

- **Holding cost :-** C_1 per unit of item per unit of time

  - Storage Cost
  - Handling Cost
  - Record keeping & Administrative cost
  - Taxes & Insurance costs

- **Shortages or penalty Cost** (Cost of back orders or shortage) C_2 Cost per unit of back order per unit of time

Inventory Decision



Terms & Notation

a) K = Set up Cost per cycle

b) C_0 = Manufacturing or purchase cost per unit

c) C_1 = Holding cost per unit per unit of time

d) D = Demand rate in units per unit of time

e) A = No. of units of item produced (or delivered) per unit of time.

f) Q = No. of unit purchase (or produce) per cycle or per order (or per run)

g) Ordering Cycle

h) t = length of cycle

I) Lead time: The time between the placement of an order and its receipt is called lead time.

Deterministic Inventory Model

- **Model 1 :- Single item, Static demand, Production model with no back ordering allowed**

(Finite and constant delivery rate during the production run)

No shortages are allowed (A>D)

Lead time is zero

S = Maximum level of inventory

s = Minimum level of inventory

Stock Policy: (s, S) policy.

Model-1……….

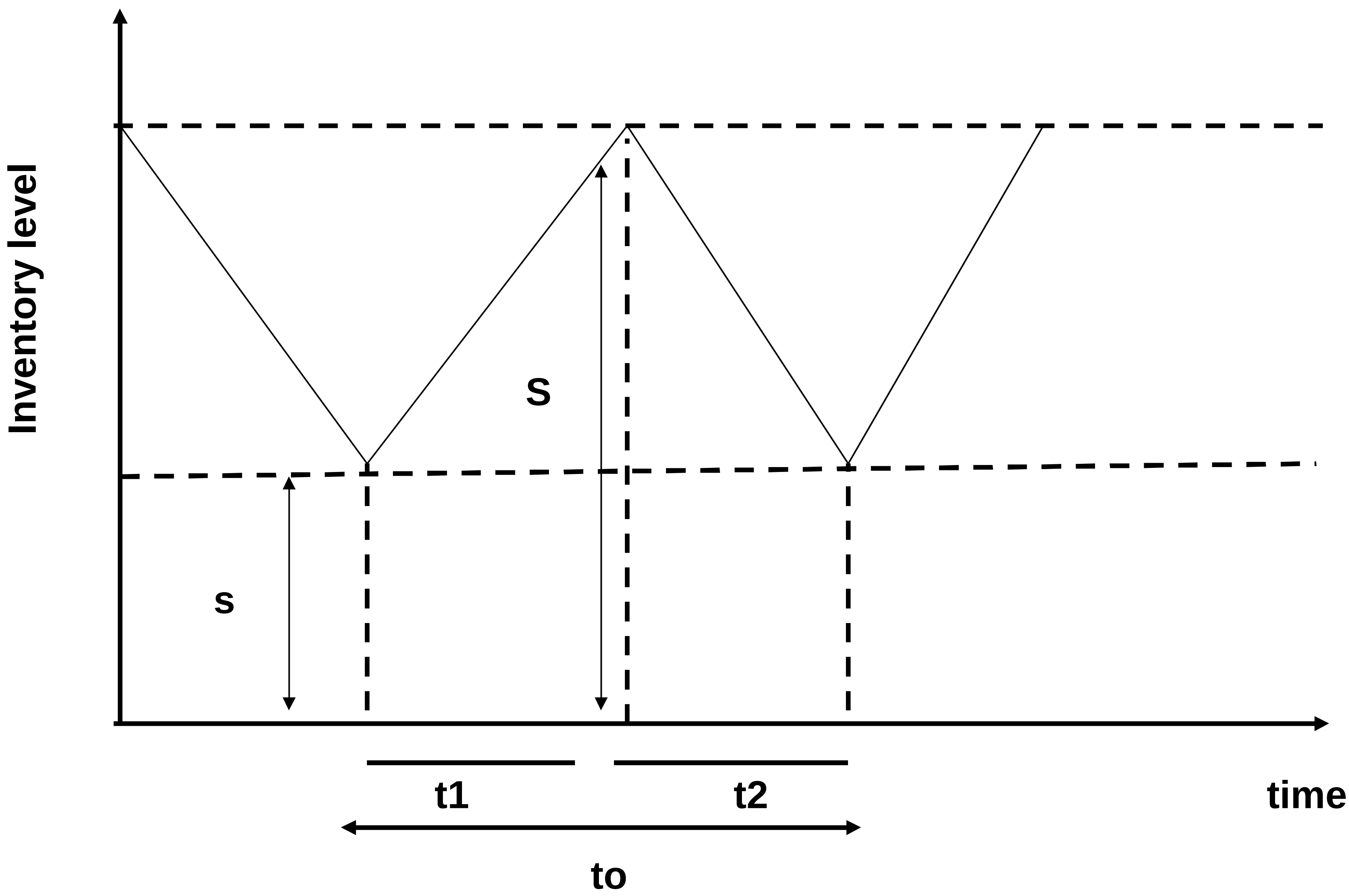
t_1 ← Time period during which items are arrived (or produced) until complete order is received. During this time period inventory build up with a rate A-D, from minimum level s to maximum level S

Model-1……

$t_2 \leftarrow$ Time period during which arrival stops & demand continues to deplete from S to s

$t_0 \leftarrow$ Length of order cycle

$$t_0=t_1+t_2$$



Inventory level
S
s
t1
t2
time
to

$$Q = At_1 = Dt_0 \text{----------- } 1$$

$$S = s + (A-D)t_1 = s + (A-D)Q/A \text{-----(from)}1$$

$$S = s + (1 - D/A)Q \text{ ---------}2$$

And

$$S - Dt_2 = s \quad \text{---------}3$$

Model-1….

Inventory holding cost = $$C1\int_{0}^{t_1} I(t)\,dt + C1\int_{t_1}^{t_0} I(t)\,dt$$

where I(t) is inventory level time t. (3)

For $[0,t_1]$, $I(t) = s + (A-D)\,t$ and for $[t_1,t_2]$, $I(t) = S - D(t-t_1)$

Substitute value in (3), Inventory holding cost per cycle is

$$=[C_1 s + 0.5\,C_1 Q(1-D/A)]\,t_0$$

Thus total cost of an order cycle is function of s and Q

i.e., $TC(Q, s) = K + C_0 Q + C_1 s t_0 + 0.5\,C_1\,t_0 Q\,(1-D/A)$

Model-1…..

Objective:- Minimize TC(Q,s)per unit time =

TCU (Q,s) =TC(Q,s) per unit of time

$= TC(Q,s)/t_0$

$= k/t_0 + C_0Q/t_0 + sC_1 + 0.5C1Q(1-D/A)$

$= C_0D+KD/Q+0.5\ C_1Q(1-D/A)+sC_1$ (4)

(Using $Q=Dt_0$)

From(4) it is obvious that for a given Q, TCU(Q,s) is minimum if s=0,putting s=0 in (4) we get

$$TCU(Q) = C_0D+KD/Q+O.5\ C_1Q(1-D/A)$$

by solving d/dQ TCU(Q)= 0,we get value of Q which minimizes TCU(Q)

Model-1…..

d/dQ TCU(Q)=0

$$Q^* = \sqrt{\frac{2kD}{C_1(1-\frac{D}{A})}} \qquad \text{...............(5)}$$

It can be seen that $d^2/dQ^2 TCU(Q)>0$ at Q=Q*

Q* is called Economic order quantity or Economic lot size (EOQ)

t_0=optimum order cycle=Q/D

$$t_o^* = Q^*/D = \sqrt{\frac{2k}{DC_1(1-\frac{D}{A})}}$$

Minimum value of TCU (Q,S)= TCU (Q*,S*)

$$= C_o D + \sqrt{2kD(1-\frac{D}{A})C_1}$$

S*, the optimum value of S is given by

S* = Q* (1-D/A) (FROM 2)

$$= \sqrt{(2kD(1-D/A))/C_1}$$

Optimal Parameters for Model-1:

(Summary of Model-1)

$$EOQ = Q^* = \sqrt{\frac{2kD}{C_1\left(1-\frac{D}{A}\right)}}$$

$$t_o^* = \sqrt{\frac{2k}{DC_1\left(1-\frac{D}{A}\right)}}$$

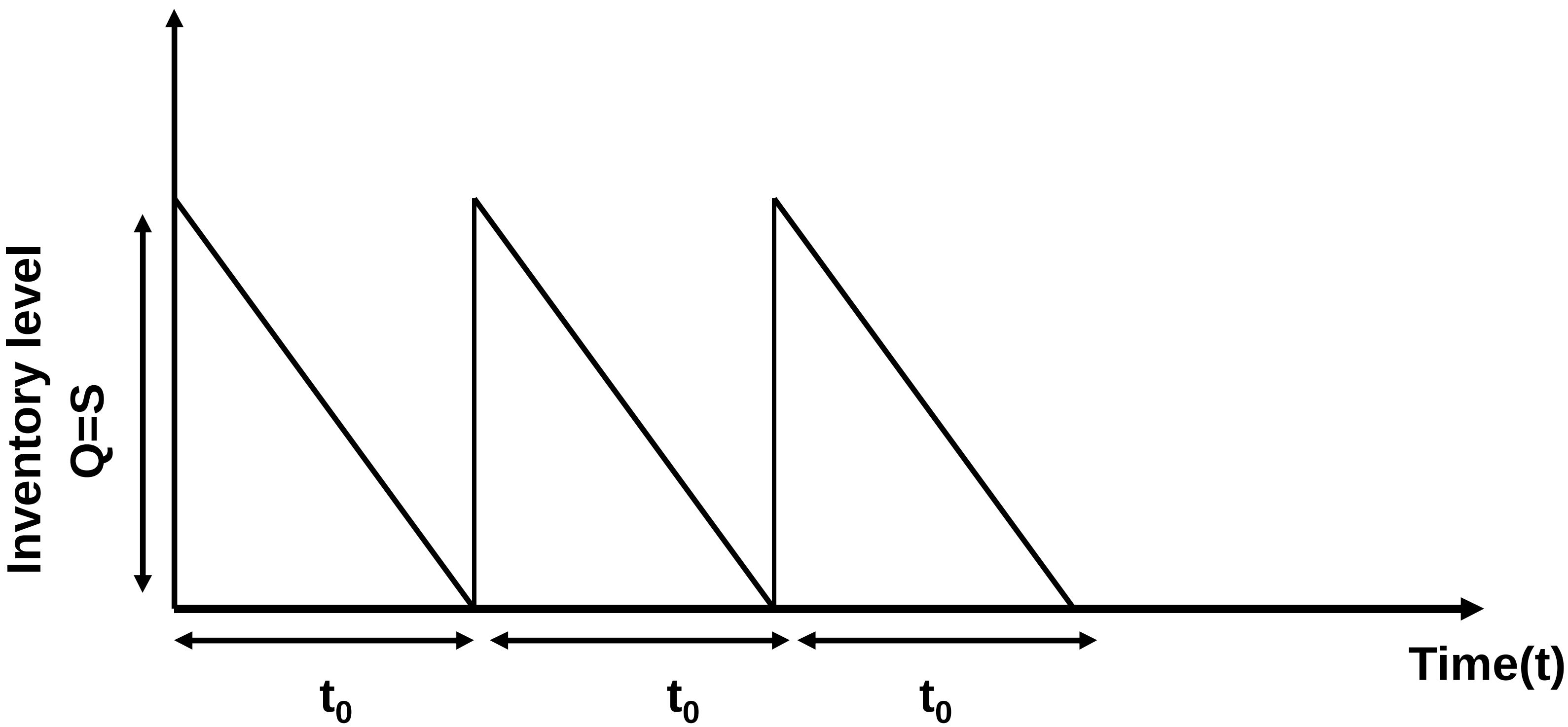
$$TCU(Q^*) = C_o D + \sqrt{2kD\left(1-\frac{D}{A}\right)C_1}$$

$$S^* = \sqrt{\frac{2kD\left(1-\frac{D}{A}\right)}{C_1}}.$$

$$s^* = 0.$$

Model II:- Single item, static Demand, purchase model with no back order, Infinite Delivery rate with no shortage allowed.

(when A is infinity in model-1)



Optimal Parameters for Model-2.

$$EOQ = Q^* = \sqrt{2kD / C_1}$$

$$t_o^* = \sqrt{2k / DC_1}$$

$$TCU(Q^*) = C_o D + \sqrt{2C_1 DK}$$

$$S^* = Q^* = \sqrt{2kD / C_1} \qquad s^* = 0.$$

Problem1

- The demand of product is 500 units per week and the delivery rate is 1000 units per week. If the purchase price is Rs 50.00 per unit, the ordering cost is Rs 100 per order, the holding cost is Rs 1.00 per unit per week. Calculate Q*, TCU(Q*).

Problem 2

- A contractor has to supply 10,000 bearings per day to an automobile manufacturer.He finds that, when he starts a production run, he can produce 25,000 bearings per day.The cost of holding a bearing in stock for one year is 20 paisa,and set up cost of a production run is Rs 180.How frequently should production run be made?

Problem 3

- You have to supply your customer 100 units of certain product every Monday(and only then).You obtain the product from a local supplier at Rs 60 per unit.The costs of ordering and transportation from the supplier are

  Rs.150per order.The cost of carrying inventory is estimated at 15 %per year of the cost of the product carried.

  (1)Describe graphically the inventory system.

  (2)Find the lot size which will minimize the cost of the system.

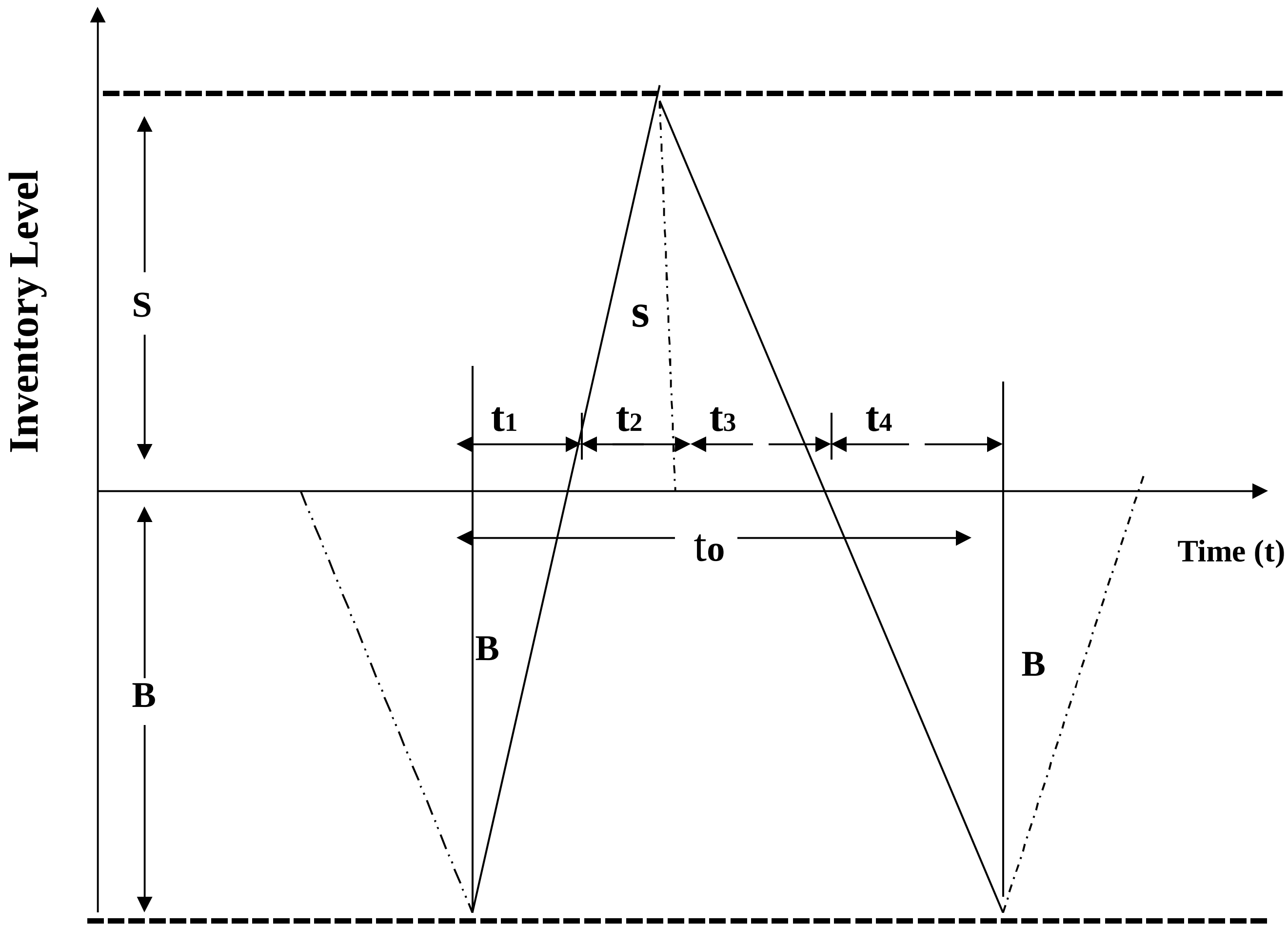
  (3)Determine the optimal cost.

Problem4

- An aircraft company uses rivets at an approximate customer rate of 2,500 kg.per year. Each unit costs Rs 30 per Kg.and the company personnel estimate that it costs Rs.130 to place an order, and that carrying cost of inventory is 10 % per year. How frequently should orders for rivets to be placed? Also determine the optimum size of each order.

MODEL-3: Shortage or Back ordering is allowed.

- Single item, static demand, production model with back ordering
- Constant delivery rate A
- Constant usage rate D
- Maximum shortage(Back order) allowed is B.
- C_2 is the back ordering cost per unit per unit of time.


Inventory Level
S
B
s
t1
t2
t3
t4
to
B
B
Time (t)

Model-3…….

- Shortages are being filled at the rate of (A-D) units per unit of time during time period t_1
- Inventory is building at a constant rate of (A-D) unit per unit of time during time period t_2
- No arrivals/delivery during time period t_3. Inventory is decreasing at a rate D per unit of time
- Shortages are building at a rate D units per unit of time during time period t_4

Observations: Model-3…..

- During t_1

$$(A-D)t_1 = B \Rightarrow t_1 = \frac{B}{A-D}. \quad \text{------------(1)}$$

- During t_2

$$(A-D)t_2 = S \Rightarrow t_2 = \frac{S}{A-D}. \quad \text{------------- (2)}$$

- During $t_{3,}$

$$Dt_3 = S \Rightarrow t_3 = \frac{S}{D}. \quad \text{------------ (3)}$$

Observations: Model-3…..

- During t_4

$$Dt_4 = B \Rightarrow t_4 = \frac{B}{D}. \qquad \text{------------(4)}$$

- Also, $Q = Dt_0 = A(t_1 + t_2).$ ------------(5)

- From equation (1),(2), and (5),

$$S = (A-D)\frac{Q}{A} - B = \left(1 - \frac{D}{A}\right)Q - B. \qquad \text{---------- (6)}$$

The period of one cycle is $t_0=t_1+t_2+t_3+t_4$

Holding Cost for one cycle

$$= C_1 \int_0^{t_2+t_3} I(t)\,dt$$

$$= C_1 \left[\int_0^{t_2} (A - D)t\,dt + \int_{t_2}^{t_2+t_3} (S - D(t - t_2))\,dt \right]$$

$$= C_1 \left[\int_0^{t_2} \frac{S}{t_2} t\,dt + \int_{t_2}^{t_2+t_3} (Dt_3 - D(t - t_2))\,dt \right]$$

$$= C_1 \left[\int_0^{t_2} \frac{S}{t_2} t\,dt + \int_{t_2}^{t_2+t_3} D(t_2 + t_3 - t)\,dt \right]$$

Holding Cost for one cycle

$$= C_1 \left[\int_0^{t_2} \frac{S}{t_2} t\,dt + \int_{t_2}^{t_2+t_3} \frac{S}{t_3}(t_2 + t_3 - t)\,dt \right]$$

$$= \frac{1}{2} C_1 S (t_2 + t_3) = \frac{1}{2} C_1 S \left(\frac{S}{A-D} + \frac{S}{D} \right).$$

$$= C_1 \frac{S^2}{2D\left(1 - \frac{D}{A}\right)} = C_1 \frac{\left\{ Q\left(1 - \frac{D}{A}\right) - B \right\}^2}{2D\left(1 - \frac{D}{A}\right)}.$$

Production Cost for one cycle $= K + C_0 Q.$

Let $B(t)$ be the level of back-order at time t.

Back order or shortage cost for one cycle

$$= C_2 \int_0^{t_1} B(t)\,dt + C_2 \int_0^{t_4} B(t)\,dt$$

$$= C_2 \int_0^{t_1} (A - D)t\,dt + C_2 \int_0^{t_4} D\,t\,dt$$

$$= C_2 \int_0^{t_1} \frac{B}{t_1} t\,dt + C_2 \int_0^{t_4} \frac{B}{t_4} t\,dt$$

$$= 0.5\,C_2 B(t_1 + t_4) = \frac{B^2 C_2}{2D\left(1 - \frac{D}{A}\right)}.$$

The Total Cost for one cycle $TC(Q, B)$

$$= K + C_0 Q + C_1 \frac{\left\{ Q\left(1 - \frac{D}{A}\right) - B \right\}^2}{2D\left(1 - \frac{D}{A}\right)} + \frac{C_2 B^2}{2D\left(1 - \frac{D}{A}\right)}.$$

The Total Cost per unit time $TCU(Q, B)$, is given by

$$TCU(Q, B) = TC(Q, B) / t_0.$$

Now, $TCU(Q,B)$

$$= \frac{KD}{Q}K + C_0 D + C_1 \frac{\left\{Q\left(1-\frac{D}{A}\right)-B\right\}^2}{2Q\left(1-\frac{D}{A}\right)} + \frac{C_2 B^2}{2Q\left(1-\frac{D}{A}\right)}.$$

Aim: Find Q and B, such that $TCU(Q,B)$ is minimum or $\min_{(Q,\,B)} TCU(Q,B)$.

$$\frac{\delta}{\delta B} TCU(Q, B) = 0$$

$$\Rightarrow B(C_1 + C_2) / Q\left(1 - \frac{D}{A}\right) - C_1 = 0.$$

$$\Rightarrow B = \frac{Q\left(1 - \frac{D}{A}\right)}{C_1 + C_2} C_1.$$

Since $\frac{\partial^2}{\partial B^2} TCU > 0 \Rightarrow TCU(Q, B)$ is minimum

when $B = \frac{Q\left(1 - \frac{D}{A}\right)}{C_1 + C_2} C_1.$

Substituting the value of B in $TCU(Q,B)$, we get

$$TCU(Q) = C_0 D + \frac{KD}{Q} + \frac{1}{2} C_1 C_2 Q\left(1 - \frac{D}{A}\right) / (C_1 + C_2).$$

Now, $TCU(Q)$ will be minimum if

$$Q = \sqrt{\left(\frac{2KD(C_1 + C_2)}{C_1 C_2\left(1 - \frac{D}{A}\right)}\right)}.$$

Optimal Parameters for Model - 3

The Economic order quantity (OOQ)

$$Q^* = \sqrt{\left(\frac{2KD(C_1 + C_2)}{C_1 C_2 \left(1 - \frac{D}{A}\right)} \right)}.$$

The Economic back order quantity (EBO)

$$B^* = \frac{Q^* \left(1 - \frac{D}{A}\right) C_1}{(C_1 + C_2)} = \sqrt{\left(\frac{2KDC_1 \left(1 - \frac{D}{A}\right)}{C_2 (C_1 + C_2)} \right)}.$$

Optimal Parameters for Model - 3.

The Optimum order cycle (OOC)

$$t_0^{\ *} = \frac{Q^*}{D} = \sqrt{\left(\frac{2K(C_1 + C_2)}{DC_1C_2\left(1 - \frac{D}{A}\right)} \right)}.$$

The optimal values of s and S of the (s, S) policy is given by

$$s^* = -B^* \text{ and } S^* = \left(1 - \frac{D}{A}\right)Q^* - B^* = \sqrt{\left(\frac{2KDC_2\left(1 - \frac{D}{A}\right)}{C_2(C_1 + C_2)} \right)}.$$

Thus the Optimum(minimum) total cost per unit of time is

$$TCU(Q^*, B^*) = C_0 D + \sqrt{\left(\frac{2KDC_1C_2\left(1 - \frac{D}{A}\right)}{C_1 + C_2} \right)}.$$

Problem6

The demand of an item is uniform @ 25 units per month.Fixed cost is Rs 15 each time a production run is made.The production cost is Rs 1=00 per item & the inventory carrying cost is Rs 0.30 per item per month.If the shortage cost is Rs 1.50 per item per month, determine how often to make a production run and of what size it should be.

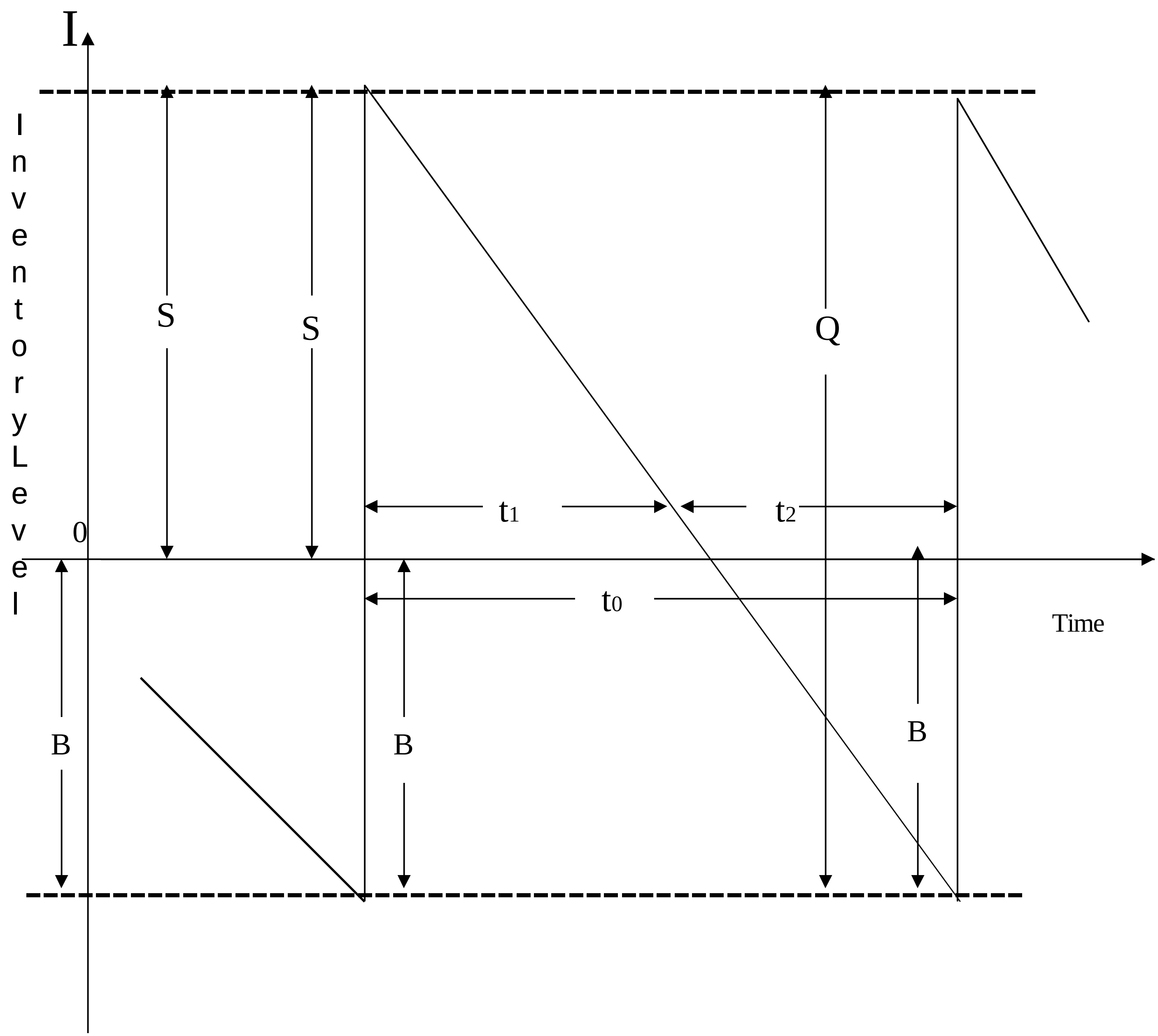
PROBLEM 7

The demand for item in a company is 18000 units per year and the company can produce the item @ 3000 per month.The cost of one set up is Rs 500.If the holding cost of one unit per month is 15 paisa, the shortage cost of 1 unit is Rs 20=00 per unit per month. Determine the optimum manufacturing quantity & no of shortages .Also determine the manufacturing time & time between setups.

Model-4:

Single item static demand inventory model with infinite delivery rate

(Just put A is infinity in model-3)


I
Inventory Level
0
S
S
Q
t_1
t_2
t_0
Time
B
B
B

Optimal Parameters for Model - 4

The Economic order quantity (OOQ)

$$Q^* = \sqrt{\left(\frac{2KD(C_1 + C_2)}{C_1 C_2}\right)}.$$

The Economic back order quantity (EBO)

$$B^* = \frac{Q^* C_1}{(C_1 + C_2)} = \sqrt{\left(\frac{2KDC_1}{C_2(C_1 + C_2)}\right)}.$$

Optimal Parameters for Model - 4.

The Optimum order cycle (OOC)

$$t_0^{\,*} = \frac{Q^*}{D} = \sqrt{\left(\frac{2K(C_1 + C_2)}{DC_1C_2}\right)}.$$

The optimal values of s and S of the (s, S) policy is given by

$$s^* = -B^* \text{ and } S^* = Q^* - B^* = \sqrt{\left(\frac{2KDC_2}{C_2(C_1 + C_2)}\right)}.$$

Thus the Optimum(minimum) total cost per unit of time is

$$TCU(Q^*, B^*) = C_0 D + \sqrt{\left(\frac{2KDC_1C_2}{C_1 + C_2}\right)}.$$

Remarks :

Model-1= Model-3 with $B = 0$ and $C_2 = \infty$.

Model-2= Model-3 with $B = 0$ and $C_2, A = \infty$.

Model-4= Model-3 with $A = \infty$.