ERROR ANALYSIS & GRAPH DRAWING

Physics Lab

(PHY F110)

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2102-B

Here you will learn about the errors!!

- Committing error is the part of an experiment.
- Estimating them is the art of experiment.
- This helps us to approach the maximum accuracy in the obtained results.

Categorically

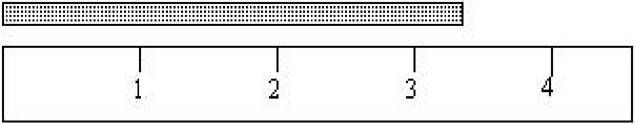
- Error Analysis
- Graphical Analysis
- Significant digits

MEASUREMENT

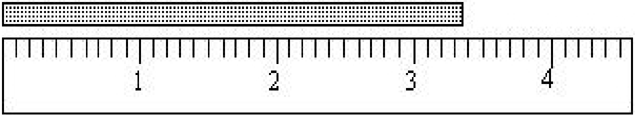
Henry I (1100-1135) who decreed that the yard should be "the distance from the tip of the King's nose to the end of his outstretched thumb".

Measuring the length of a rod with rulers

On a ruler with a coarse scale, the rod is between 3 and 4 cm, and we estimate it to be about 3.3 cm. The instrument least count is 1 cm.



On a ruler with a finer scale the rod is between 3.3 and 3.4 cm, and we estimate it to be about 3.38 cm. Instrument least count is 0.1 cm.



Error due to calculations

Error in the primary measurements cause uncertainty in the final measurement.

Errors in the measurement

$$g = \frac{4\pi^2 L}{T^2}$$

Error in measuring L and T will add up to the final result.

NOMENCLATURE OF ERRORS

- Blunder
- Systematic error
- Random error

BLUNDERS

- Experimenter makes a genuine mistake in reading an instrument wrongly.
- This can be avoided by taking large number of data points, discarding an entirely different value.

SYSTEMATIC ERRORS

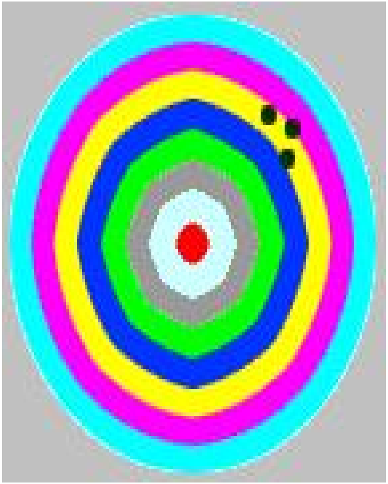
- This is an instrumental error.
- Constant error which occurs all the time.
- Difficult to detect.
- Examples:The scale itself is incorrect; causes error in length measurement.
- Calibration of the instruments should be done to avoid the errors.

RANDOM ERRORS

- Caused by unknown and unpredictable changes in the experiment and in the instrument.
- By repeating the experiments to large number of times one can minimize this (statistical analysis).
- Difficult to be eliminated. Only one can estimate it.

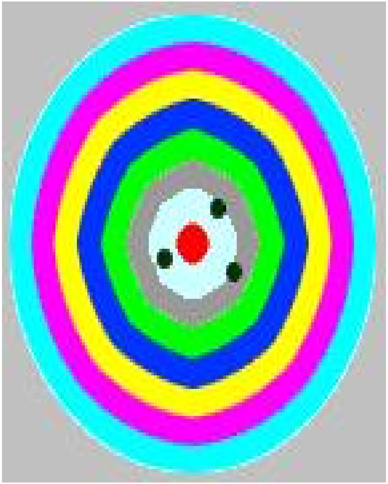
Precision

- Random error is small, precision is high.
- High precision means minimum random error.



Precision

- Accuracy means the best possible mesurement (the true value).
- High accuracy means minimum systematic error.

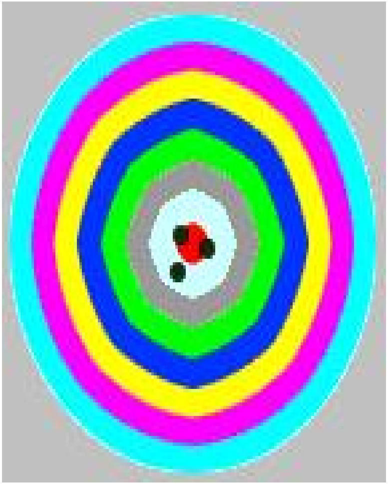


Accuracy & Precision

- Higher the accuracy; minimum the systematic error.
- Higher the precision; minimum the random error.

Accuracy with Precision

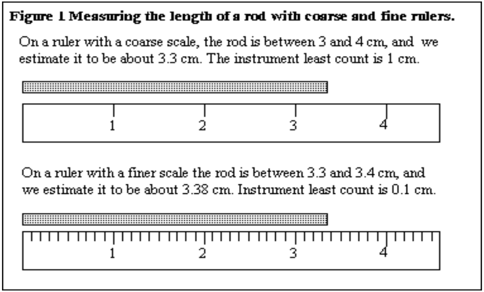
Measurement with high accuracy and precision is highly RELIABLE !!



Estimation of Errors: Least count

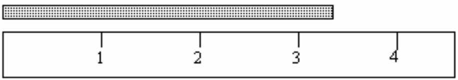
Maximum error in any primary measurement

- Instrument least count
- Effective least count

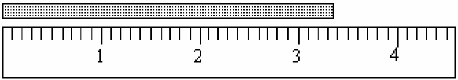


- Effective least count appears as = 0.3 cm
- To be in the safer side Length = 3.3 cm

Estimation of Errors: Least count



Effective Length $=$ 3.3 cm



Effective Length $=$ 3.35 cm

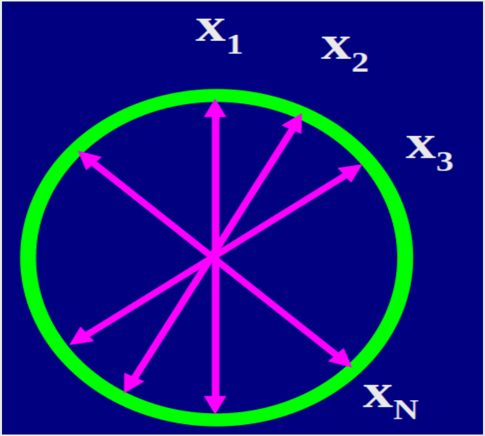
- Always go for high least count apparatus.
- Higher the Accuracy, systematic error is minimum.

Error analysis (Statistical)

- One data point measurement.
- One variable measurement.
- Two variable measurement.

One Variable measurement

Diameter of a ball



One Variable measurement

Diameter of a ball: Mean Value

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Diameter of a ball: Standard Deviation

$$\sigma(\Delta x) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

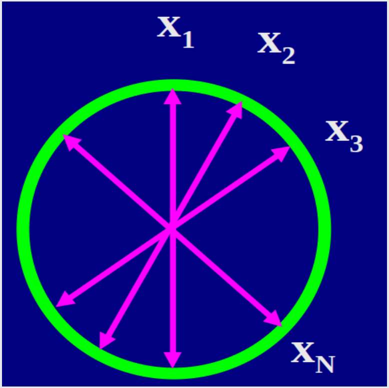
One Variable measurement

Diameter of a ball: Mean Value

- Measure of dispersion of set of values.
- Defined as root mean square deviation from the mean value.
- If many data points are close to mean SD is small.
- Data points are equal to means, $\mathrm{SD} = 0$.

One Variable measurement

Diameter of a ball $= (x \pm \Delta x)$ units



Linear fit of data between two variable

Linear fit

Let there are N points of measurements

$(x_1, y_1),\ (x_2, y_2)...(x_N, y_N)$

$$\boxed{y = mx + c}$$

where the measured/calculated values are x and y while, the slope m and the intercept, c you will obtain.

Linear fit of data between two variable

For the best fit line, the quantity S

$$S = \sum_i (y_i - mx_i - c)^2$$

should be minimum.

Thus, we can write,

$$\frac{\partial S}{\partial m} = -2\sum_i (y_i - mx_i - c)x_i = 0$$

$$\frac{\partial S}{\partial c} = -2\sum_i (y_i - mx_i - c) = 0$$

Linear fit of data between two variable

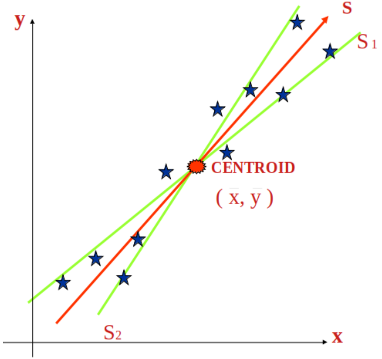
Thus, the slope and intercept will be,

$$m = \frac{\sum(x_i - \bar{x}) * y_i}{\sum(x_i - \bar{x})^2}$$

$$c = \bar{y} - m\bar{x}$$

Graphical Method for Best Fit Line

- Plot all the data points.
- Plot centroid (x, y).
- Draw limiting lines (S1 and S2).
- Draw a best fit line (S)
- Get $\Delta S = (S_1 \sim S_2)/2$.



Propagation of Errors

If Δf is difference in a quantity $f(x, y, z)$ from accurate value then,

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2}$$

for small changes in $x,\ y,\ \& z$.

Propagation of Errors: Rule-1

Addition/Subtraction

$$Z = X + Y$$

or,

$$Z = X - Y$$

The error(uncertainty) will be

$$\Delta Z = \sqrt{(\Delta X)^2 + (\Delta Y)^2}$$

Propagation of Errors: Rule-1

An example

Suppose we have measured the starting position as $x_1 = 9.3 \pm 0.2$ m and the finishing position as $x_2 = 14.4 \pm 0.3$ m. Then the displacement is, $d = x_2 - x_1 = (14.4 - 9.3)$ m = 5.1 m. The error (uncertainty) in the displacement is

$$\sqrt{0.2^2 + 0.3^2} \text{ m} = 0.36 \text{ m}$$

The result will be quoted as, 5.1 ± 0.36 m.

Propagation of Errors: Rule-2

Addition/Subtraction

$$Z = X \times Y \quad \text{or} \quad Z = \frac{X}{Y}$$

The error(uncertainty) will be

$$\frac{\Delta Z}{Z} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2}$$

Propagation of Errors: Rule-2

An example

$$g = \frac{4\pi^2 L}{T^2}; \qquad \frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$$

Propagation of Errors: An example

We have measured a displacement of as 5.1 ± 0.4 m in time 0.4 ± 0.1 s. What is the measured velocity and the error(uncertainty) in the velocity?

$$v = \frac{5.1}{0.4} = 12.75 \text{ m/s}$$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta t}{t}\right)^2} = 3.34 \text{ m/s}$$

The result will be quoted as, 12.75 ± 3.34 m.

Propagation of Errors: Rule-3

An example

$$Z = X^n; \qquad \frac{\Delta Z}{Z} = n\frac{\Delta X}{X}$$

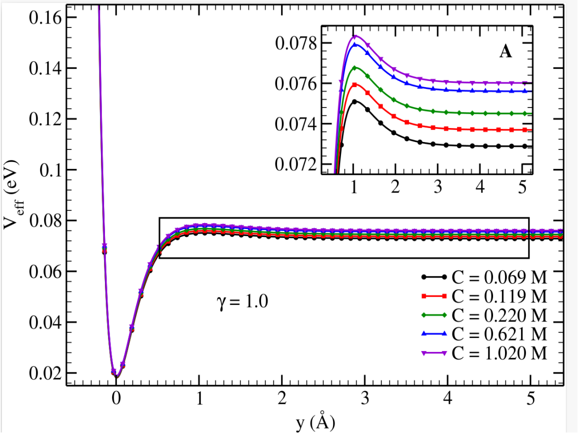
An example: Error in volume of a sphere

$$V = \frac{4}{3}\pi R^3; \qquad \frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

Plotting a Graph: Some Advisory

- Use Sharp Pencil to draw the graph.
- Draw on full page of graph paper and use appropriate scale.
- Plot dependent variable on the vertical y-axis and the independent variable on the x-axis.
- Label the axes and the data points properly.
- Title the graph.
- Indicate error bars.

An Example Graph



Significant Figures

The digits required to express a number to the same accuracy as the measurement it represents are known as significant figures.

Understand the difference between 1 and 1.00

Number	Significant digits
22	2
0.046	2

Least Count

How to find out least count of an instrument?

$$L.C. = \frac{\text{Smallest Main Scale Reading}}{\text{Total Number of Vernier division}}$$

More in class!