LABORATORY MANUAL

FOR

PHYSICS LABORATORY - I

PHY F110

by

Department of Physics

BITS, Pilani

Educational Development Division

Birla Institute of Technology & Science

Pilani (Rajasthan), India

2013

Preface

It gives us great pleasure to bring out this fourth revised version of the laboratory manual for Physics Laboratory I, earlier known as Measurement Techniques I.

The first revised version of the manual was brought out in August 2001, after rewriting some of the experiments in the older version, deleting some that were found unsuitable, and adding a few new experiments. The second revision of the manual was done in 2007, in which certain errors were corrected and a circuit diagram was redrawn. In the third revision, done in 2009, the data sets for the first experiment, Error Analysis and graph Plotting, were completely changed. In the fourth revision, we have modified certain experiments and further revised the manual keeping in view the fact that all calculations and graph plotting are now done by students using computers in the lab.

We are happy to acknowledge the continuous encouragement we have received from the institute administration in upgrading and modernizing the lab. In particular, we would like to acknowledge the generous financial support of the institute for buying a number of new experimental sets and desktop computers for each such set. We are also happy to acknowledge the financial support of UGC under Infrastructure Grant, in this modernization process.

This revision of the manual was done after thorough discussion among the members of the Physics Department. We trust such discussions will continue to happen and will lead to continual improvement of the laboratory and the manual.

Faculty Members

Department of Physics

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CONTENTS

Instructions for Laboratory

General

- The objective of the laboratory is *learning*. The experiments are designed to illustrate phenomena in different areas of Physics and to expose you to measuring instruments. Conduct the experiments with interest and an attitude of learning.
- Be prompt in arriving to the laboratory and come well prepared for the experiment.
- Work quietly and carefully (the whole purpose of experimentation is to make reliable measurements!) and equally share the work with your partners.
- Be totally honest in recording and representing your data. Never make up readings or doctor them to get a better fit for a graph. If a particular reading appears wrong repeat the measurement carefully. In any event all the data recorded in the tables have to be faithfully displayed on the graph.
- All presentations of data, tables, graphs and calculations should be *neatly* and *carefully* done.

Specific

- Every student has to have his/her individual copy of the manual. The laboratory manual has to be brought to the laboratory when you come to do the experiment.
- All the measurements have to be neatly recorded in the manual itself. The readings entered in the manual have to verify by your instructor before you leave the laboratory.
- Necessary graph papers are provided in the manual at the end of each of experiment. Learn to optimize on usage of graph paper. For example, in Experiment 10 (Plank's Constant) you do not need three separate sheets to represent the graphs of lnI_{ph} vs. T^{-1} for the three different filters. All the three graphs can be accommodated on a single graph sheet.
- Graphs should be neatly drawn with *pencil.* Always *label* graphs and the axes and display units.
- The duration of the laboratory is for 2 hours. You are supposed to be fully engaged for the complete duration of the laboratory. *You cannot leave the laboratory until the completion of the laboratory class hours.*

- **Every time switch off the setup before leave**, if you finish early, spend the remaining time to complete the calculations and drawing graphs. Come equipped with calculators, scales, pencils etc.
- Please do not fiddle idly with apparatus. Handle instruments with care. Report any breakage to the Instructor. Return all the equipment you have signed out for the purpose of your experiment.

Bibliography

Here is a short list of reference books which may be useful for further reading in physics or instrumentation relevant to the experiments. Also included are some reference books of general interest with regard to science and experimentation.

1. "Fundamentals of Physics", *D. Haliday, R. Resnick and J. Walker,* John Wiley and Sons, Inc. New York, 2001.
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Error Analysis and Graph Drawing

I. Introduction:

I.1 It is impossible to do an experimental measurement with perfect accuracy. There is always an uncertainty associated with any measured quantity in an experiment even in the most carefully done experiment and despite using the most sophisticated instruments. This uncertainty in the measured value is known as the error in that particular measured quantity. There is no way by which one can measure a quantity with one hundred percent accuracy. In presenting experimental results it is very important to objectively estimate the error in the measured result. Such an exercise is very basic to experimental science. The importance of characterizing the accuracy and reliability of an experimental result is difficult to understate when we keep in mind that it is experimental evidence that validate scientific theories. Likewise, reliability and accuracy of measurements are also deeply relevant to Engineering.

The complete science of error analysis involves the theory of statistics (see Ref. 1,2) and is too involved to present here. This short presentation is intended to introduce the student to some basic aspects of error analysis and graph drawing, which it is expected that the student will then put into practice when presenting his/her results of the coming experiments.

I.2 When a measurement of a physical quantity is repeated, the results of the various measurements will, in general, spread over a range of values. This spread in the measured results are due to the errors in the experiment. Errors are generally classified into two types: *systematic* (or *determinate*) errors and *random* (or *indeterminate*) errors. A systematic error is an error, which is constant throughout a set of readings. Systematic errors lead to a clustering of the measured values around a value displaced from the "true" value of the quantity. Random errors on the other hand, can be either positive or negative and lead to a dispersion of the measurements around a mean value. For example, in a time period measurement, errors in starting and stopping the clock will lead to random errors, while a defect in the working of the clock will lead to systematic error. A striking example of systematic error is the measurement of the value of the electric charge of the electron '*e*' by Millikan by his Oil Drop method. Millikan underestimated the viscosity of air, leading to a lower value for his result

$$e = (1.591 \pm 0.002) \times 10^{-19} \text{ C.} \qquad (1)$$

Compare this with a more modern and accurate value (Cohen and Taylor 1973, Ref 3)

$$e = (1.602\ 189 \pm 0.000\ 005) \times 10^{-19} \text{ C.} \qquad (2)$$

Systematic errors need to be carefully uncovered for the particular experimental set-up and eliminated by correcting the results of the measurements.

I.3 Random errors are handled using statistical analysis. Assume that a large number (N) of measurements are taken of a quantity Q giving values $Q_1, Q_2, Q_3, \ldots Q_N$. Let $\overline{Q}$ be the mean value of these measurements

$$\overline{Q} = \frac{1}{N}\sum Q_i, i = 1,2,.....N \qquad (3)$$

and let 'd' be the deviation in the measurements

$$d = \sqrt{\frac{1}{N}\sum\left(Q_i - \overline{Q}\right)^2}, i = 1,2,.....N\ . \qquad (4)$$

The result of the measurement is quoted (assuming systematic errors have been eliminated) as

$$Q = \overline{Q} \pm d. \qquad (5)$$

The error ΔQ in the quantity Q is then taken to be the deviation d. (This is called the *standard error* in Q).

In a single measurement of a physical quantity, the error can be estimated as the least count (or its fraction) of the instrument being used.

As an example, the result of a measurement of the radius of curvature *R*, of a plaino-convex could be quoted as

$$R = 140 \pm\ 0.2 \text{ cm.} \tag{6}$$

This means that we expect that the value of *R* to be in the range 139.8 to 140.2 cm. Note however, that this does not mean that the true value of *R* necessarily lies in this range, only that there is a probability that it will do so.

The error in a measurement can also be quoted as a percent error,

$$\frac{\Delta Q}{\overline{\overline{Q}}} \times 100 = \frac{d}{\overline{\overline{Q}}} \times 100\,. \tag{7}$$

For example, the percent error in *R* is 0.143%.

I.4 **Combination of errors:**

Often the value of a quantity of interest may depend on other measured quantities. For example we could have a quantity *Q* which is a function *F* of a number of independent (actively controlled by us) variables say *x*,*y* and *z* i.e.,

$$Q = Q(x,y,z) \tag{8}$$

In general, the error in *Q* is related to errors in *x, y, z, …,* as follows (for small errors)

$$(\Delta Q)^2 = (\Delta Q_x)^2 + (\Delta Q_y)^2 + (\Delta Q_z)^2 + \ldots \tag{9}$$

where

$$\Delta Q_x = \left(\frac{\partial Q}{\partial x}\right)\Delta x \quad ; \quad \Delta Q_y = \left(\frac{\partial Q}{\partial y}\right)\Delta y \quad ; \quad \Delta Q_z = \left(\frac{\partial Q}{\partial z}\right)\Delta z \quad etc.$$

The following table summarizes the results for combining errors for some standard functions. Try to derive some of these results.

Sr No	Function *Q(x,y)*	Error *ΔQ* or Fractional Error *ΔQ/Q*
1	$Q = x + y$	$\Delta Q = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
2	$Q = x - y$	$\Delta Q = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
3	$Q = x\,y$	$\left(\frac{\Delta Q}{\overline{Q}}\right)^2 = \left(\frac{\Delta x}{\overline{x}}\right)^2 + \left(\frac{\Delta y}{\overline{y}}\right)^2 \Rightarrow \left(\frac{\Delta Q}{\overline{Q}}\right) = \sqrt{\left(\frac{\Delta x}{\overline{x}}\right)^2 + \left(\frac{\Delta y}{\overline{y}}\right)^2}$
4	$Q = x/y$	$\left(\frac{\Delta Q}{\overline{Q}}\right)^2 = \left(\frac{\Delta x}{\overline{x}}\right)^2 + \left(\frac{\Delta y}{\overline{y}}\right)^2 \Rightarrow \left(\frac{\Delta Q}{\overline{Q}}\right) = \sqrt{\left(\frac{\Delta x}{\overline{x}}\right)^2 + \left(\frac{\Delta y}{\overline{y}}\right)^2}$
5	$Q = x^n$	$\frac{\Delta Q}{\overline{Q}} = n\frac{\Delta x}{\overline{x}}$
6	$Q = \ln x$	$\Delta Q = \frac{\Delta x}{\overline{x}}$
7	$\mathbf{Q} = \mathrm{e}^x$	$\frac{\Delta Q}{\overline{Q}} = \Delta x$

II Drawing of best fit straight line graph:

Below we describe how to fit a straight line to a set of data. Relations that are not linear can be transformed to a linear one by an appropriate transformation of the variables (as you will learn from these assignments).

To draw the best fit straight line graph through a set of scattered experimental data points we will follow a standard statistical method, known as *least squares fit* method.

Let us consider a set of N experimental data points (x_1,y_1), (x_2,y_2), $(x_3,y_3)\ldots(x_N,y_N)$. It is well known that a straight line graph is described by the equation

$$y = mx + c. \qquad (10)$$

We ask the question: how are the slope m and the y-intercept c to be determined such that a straight line best approximates the curve passing through the data points? Let $S_i = y_i - mx_i - c$ be the deviation of any experimental point $P(x_i, y_i)$, from the best fit line. Then, the gradient 'm' and the intercept 'c' of the best fit straight line has to be found such that the quantity

$$S = \sum_i (y_i - mx_i - c)^2$$

is a minimum. We thus require

$$\frac{\partial S}{\partial m} = -2\sum_i x_i(y_i - mx_i - c) = 0 \quad and \quad \frac{\partial S}{\partial c} = -2\sum_i (y_i - mx_i - c) = 0,$$

which give,

$$m\sum x_{i^2} + c\sum x_i = \sum x_i y_i \quad and \quad m\sum x_i + Nc = \sum y_i.$$

The second equation can be rewritten as $\bar{y} = m\bar{x} + c$, where $\bar{y} = \frac{1}{N}\sum y_i$ *and* $\bar{x} = \left(\frac{1}{N}\sum x_i\right)$ showing that the best fit straight line passes through the centroid $(\bar{x}, \bar{y})$ of the points (x_i,y_i). The required values of m and c can be calculated from the above two equations to be

$$m = \frac{\sum(x_i - \bar{x})y_i}{\sum(x_i - \bar{x})^2} \quad and \quad c = \bar{y} - m\bar{x}\,. \tag{11}$$

The best fit straight line can be drawn by calculating m and c from above. A graphical method of obtaining the best fit line is to rotate a transparent ruler about the centroid so that it passes through the clusters of points at the top right and at the bottom left. This line will give the maximum error in m, $(\Delta m)_1$ on one side. Do the same to find out the maximum error in m, $(\Delta m)_2$ on the other side. Now bisect the angle between these two lines and that will be the best fit line through the experimental data.

What are the errors in the gradient and intercept due to errors in the experimental data points? The estimates of the standard errors in the slope and intercept are

$$(\Delta m)^2 \approx \frac{1}{D}\frac{\sum S_i{}^2}{N-2} \quad and \quad (\Delta c)^2 \approx \left(\frac{1}{N} + \frac{\bar{x}^2}{D}\right)\frac{\sum S_i{}^2}{N-2}\,,$$

where $D = \sum(x_i - \bar{x})^2$ and S_i is the deviation $S_i = y_i - mx_i - c$.

II.1 Presentation of error associated with experimental data in a graph:

Let us consider a function, $y = f(x)$, where x is an independent parameter which in the hand of the experimentalist during performing the experiments and y is the experimental data which is having a value depending upon the x and the instruments. Let the error associated with x be $\pm\Delta \underline{x}$ and that for y be $\pm\Delta y$. One can represent $\pm\Delta x$ and $\pm\Delta y$ with the experimental data point $P(x,y)$ on the graph paper. To do that, first plot $P(x,y)$ on the graph paper, then draw a vertical line parallel to y axis about the point $P(x,y)$ of length $2\Delta y$. So upper half of the line represents the error $+\Delta y$ and the lower half represents $-\Delta y$ error. To present $\pm\Delta x$, draw horizontal lines at the two ends of the vertical line of length $2\Delta x$ each. The whole presentation is now giving the errors associated with the experimental point $P(x,y)$.

Figure 1 is an example of experimental data of resonance absorption of γ-ray experiment (Mössbauer spectroscopy) with error associated with each experimental data. The solid line gives the fitted curve through the experimental data. Note that the error in the variable along horizontal axis is not shown.

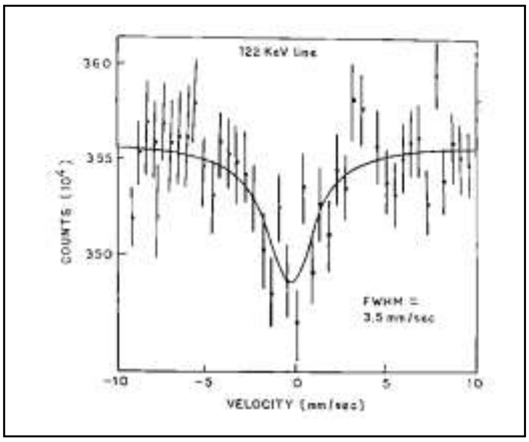

Fig.1

II.3 Use of graphs in experimental physics:

In practical physics, the graph of the experimental data is most important in improving the understanding of the experimental results. Moreover from the graphs one can calculate unknowns related to the experiments and one can compare the experimental data with the theoretical curve when they are presented on same graph. There are different types of graph papers available in market. So, one should choose the appropriate type of graph paper to present their experimental results in the best way depending upon the values of the experimental data and the theoretical expression of the functions. To understand all those some of the assignments are given below in addition to those we discussed before.

III **Using calculator to fit curves**

After doing these assignments you also have to verify the best fit parameters, you have obtained, by using your calculator. Calculators do the kind of curve fitting we want. Find out the procedure, specific to your calculator, to fit curves to a set of data. You will find it in the calculator's manual under heading like "statistical" and sub heading "Regression". If the formula of interest is one of the standard relations in the calculator then you simply enter the data and read of the best fit parameters that the calculator responds with. For other formulae you can use the trick of transforming to a new set of variables. In all your future work you can do the regression analysis using a calculator.

IV. Exercises and Viva Questions

1. What is the general classification of errors? Give an example of each. How are they taken care of?

Radius of curvature (cm)
130.121
130.136
130.139
130.148
130.155
130.162
130.169

2. What is the meaning of standard error? Calculate the standard error for the hypothetical data given in the adjacent table. Express the quantity as in eq. (5), i.e. $R = \overline{R} \pm d$.
3. What is the percent error in Millikan's measurement of the charge of the electron: $e = (1.591 \pm 0.002) \times 10^{-19}$ C?
4. What is the error in the volume of a cube $V = L^3$ if the error in L is 0.01m? If L is measured as $L = 2 \pm 0.01$, express the value of V in a similar manner.
5. A small steel ball-bearing rests on top of a horizontal table. The radius (R) of the ball is measured using a micrometer screw gauge (with vernier least count 0.05 mm) to be 2.15 mm. The height of the table is found using an ordinary meter scale to be 90 cm. What is the height of the center of the steel ball from the floor (include the error)?
6. Let $Q = x - y$, where $x = 100 \pm 2$ and $y = 96 \pm 2$. Calculate Q (express the result with the error included).
7. Consider the quantity $Q = x/y$. If $x = 50 \pm 1$ and $y = 3 \pm 0.2$. Calculate Q (express the result with the error included).
8. In an experiment involving diffraction of sodium light using a diffraction grating, the doublet lines are unresolved at first order and a single spectral line is seen at an angle of 13^o. If the least count of the vernier of the telescope is 1′, what will be the error in the calculated value of the grating constant d? (Principal maxima of a grating occur at angles θ such that $d sin\theta = m\lambda$. The wavelength separation between the sodium doublet lines is 6 A^o).
9. Consider an experiment to measure the gravitational acceleration g by measuring the time period of a simple pendulum. What are the possible sources of systematic error in this experiment?

10. “If there are always errors in any measurement then there is nothing like the ‘true’ value of any measured quantity”. Comment on this statement. In what sense then do you understand the values of ‘physical constants’ to be constants?

References:

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Experiment 1

Coupled Pendulums

Apparatus:

Two compound pendulums, coupling spring, convergent lens, filament bulb on stand, screen on stand, stop clock.

Purpose of experiment:

To study normal modes of oscillation of two coupled pendulums and to measure the normal mode frequencies.

Basic methodology:

Two identical compound pendulums are coupled by means of a spring. Normal mode oscillations are excited and their frequencies are measured.

I. Introduction

I.1 The reason why the study of simple harmonic motion is important is the very general manner in which such a motion arises when we want the response of a system to small deviations from the equilibrium configuration. This happens for a wide variety of systems in Physics and Engineering.

The response of a system to small deformations can usually be described in terms of individual oscillators making up the system. However, the oscillators will not have independent motion but are generally coupled to other oscillators. Think for example of vibrations in a solid. A solid can be thought of as being composed of a lattice of atoms connected to each other by springs. The motion of each individual atom is coupled to that of its neighbouring atoms.

The description of a system of coupled oscillators can be done in terms of its **normal modes**. In a coupled system the individual oscillators may have different natural frequencies. A normal mode motion of the system however will be one in which all the individual oscillators oscillate with the same frequency (called the normal mode frequency) and with definite phase relations between the individual motions. If a system has *n* degrees of freedom (i.e. has *n* coupled oscillators) then there will be n normal modes of the system. A general disturbance of the system can be described in terms of a superposition of normal mode vibrations. If a single oscillator is excited, then eventually the energy gets transferred to all the modes.

I.2 In this experiment we will study some of the above features in the simple case of two coupled compound pendulums. The system studied in the experiment consists of two identical rigid pendulums, *A* and *B*. A linear spring couples the oscillations of the two pendulums. A schematic diagram of the system is given in Figure 1.

The motion of the two pendulums *A* and *B* can be modeled by the following coupled differential equations (θ_A and θ_B are the angular displacements of *A* and *B*, *I* their moments of inertia)

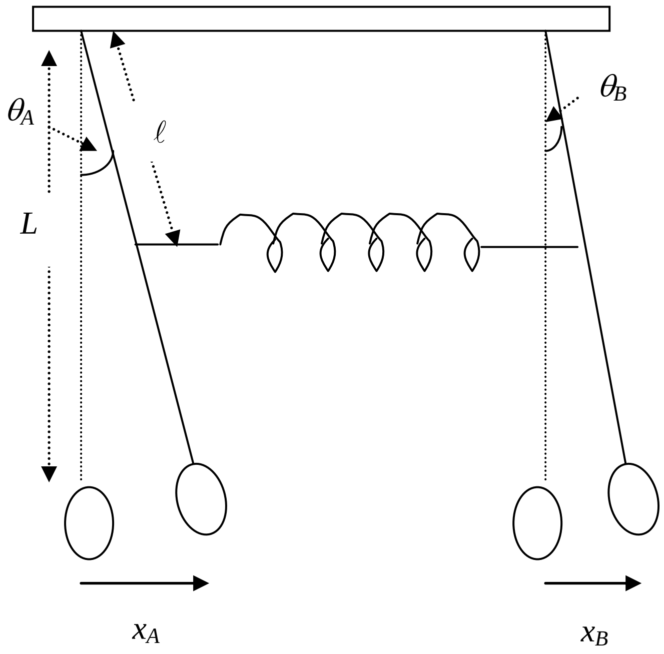


Fig.1

The equations of motion of the two physical pendulums are easily obtained. Let θ_A and θ_B be the angular displacements, and x_A and x_B the linear displacements of the two pendulums respectively. The compression of the spring will be $(x_A - x_B)\frac{\ell}{L}$ where ℓ is the distance between the point of suspension and the point where the spring is attached and L the length of the pendulum. The rotational equation for pendulum. A will thus be

$$I\frac{d^2\theta_A}{dt^2} = -mgL_{CM}\sin\theta_A - k(x_A - x_B)\frac{\ell}{L}\ell\cos\theta_A, \quad (1)$$

where, the first term on the right is the restoring torque due to gravity (L_{CM} being the distance between the point of suspension and the position of the center of mass of pendulum *A*) while the second term that due to the spring force. Assuming the mass attached to pendulum *A* to be sufficiently heavy we can equate L_{CM} and L. We also consider small displacements θ_A, so that $\sin\theta_A \approx \theta_A$ and $\cos\theta_A \approx 1$. Substituting $\theta_A = x_A/L$ and using the above approximations, we obtain the following equation of motion for the linear displacement x_A:

$$\frac{d^2x_A}{dt^2} = -\left(\frac{mgL}{I}\right)x_A - k(x_A - x_B)\frac{\ell^2}{I} \quad (2)$$

Likewise the equation for x_B is

$$\frac{d^2x_B}{dt^2} = -\left(\frac{mgL}{I}\right)x_B + k(x_A - x_B)\frac{\ell^2}{I} \quad (3)$$

Equations (2) and (3) are **coupled**, i.e. the equation for x_A involves x_B and vice-versa. Without the coupling, i.e. in the absence of the spring, x_A and x_B would be independent oscillations with the natural frequency $\omega_0^2 = \sqrt{\frac{mgL}{I}}$.

I.3 It is easy to find uncoupled equations describing the normal modes of the system. Define the variables

$$x_1 = x_A + x_B \quad ; \quad x_2 = x_A - x_B \tag{4}$$

Adding and subtracting eqs. (2) and (3) we obtain equations for the variables x_1 and x_2 as

$$\frac{d^2 x_1}{dt^2} = -\left(\frac{mgL}{I}\right)x_1 \tag{5}$$

$$\frac{d^2 x_2}{dt^2} = -\left(\frac{mgL}{I}\right)x_2 - 2\frac{k\ell^2}{I}x_2 \tag{6}$$

Note that the equations for x_1 and x_2 are uncoupled. The variables x_1 and x_2 describe independent oscillations and are the two normal modes of the system. The general solution to these equations will be

$$x_1(t) = A_1 \cos(\omega_1 t + \varphi_1) \quad ; \quad x_2(t) = A_2 \cos(\omega_2 t + \varphi_2) \tag{7}$$

(A_1, A_2 being the amplitudes of the two modes and φ_1, φ_2 arbitrary phases). The corresponding natural frequencies are the normal mode frequencies:

$$\omega_1 = \omega_0 \quad ; \quad \omega_2 = \sqrt{\omega_0^2 + \frac{2k\ell^2}{I}} = \omega_0\sqrt{1 + \frac{2k\ell^2}{mgL}} \tag{8}$$

where $\omega_0 = \sqrt{\frac{mgL}{I}}$ is the natural frequency of each uncoupled pendulum.

It is instructive to visualize the motion of the coupled system in these normal modes. If we excite only the first normal mode, i.e. $x_1(t) \neq 0$, but $x_2(t) = 0$ at all times, the individual motions of pendulums A and B will be

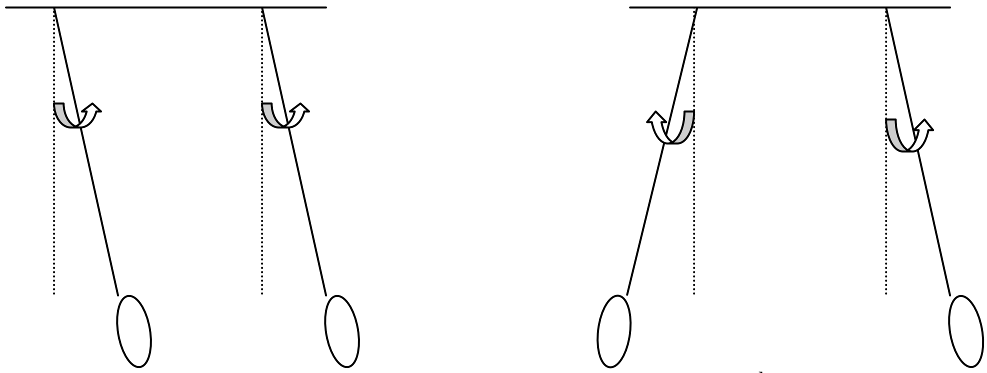
$$x_A(t) = \frac{1}{2}\left(x_1(t) + x_2(t)\right) = \frac{A_1}{2}\cos\left(\omega_1 t + \varphi_1\right) = x_B(t) = \frac{1}{2}\left(x_1(t) - x_2(t)\right) \tag{9}$$

Note that in this mode $x_A = x_B$. This describes a motion in which both pendulums move in phase with the same displacement and with frequency ω_1.

On the other hand if the second mode is excited, i.e. $x_1(t)=0$ for all times and $x_2(t) \neq 0$ the individual motions are

$$x_A(t) = \frac{1}{2}\left(x_1(t) + x_2(t)\right) = \frac{A_2}{2}\cos\left(\omega_2 t + \varphi_2\right) = -x_B(t) = \frac{-1}{2}\left(x_1(t) - x_2(t)\right) \tag{10}$$

In this mode the displacements of the pendulums are always opposite ($x_A(t) = -x_B(t)$). Their motions have the same amplitude and frequency ($=\omega_2$) but with a relative phase difference of π. Figure 2 shows the motions in the normal modes.



1^{st} normal mode 2^{nd} normal mode

Fig.2

I.4 A general motion of the coupled pendulums will be a superposition of the motions of the two normal modes:

$$x_A(t) = \frac{1}{2}\left[A_1 cos\left(\omega_1 t + \varphi_1\right) + A_2 cos\left(\omega_2 t + \varphi_2\right)\right]$$

$$x_B(t) = \frac{1}{2}\left[A_1 cos\left(\omega_1 t + \varphi_1\right) - A_2 cos\left(\omega_2 t + \varphi_2\right)\right] \tag{11}$$

For a given initial condition the unknown constants (two amplitudes and two phases) can be solved. Consider the case where the pendulum A is lifted to a displacement A at t = 0 and released from rest while B remains at its equilibrium position at t = 0. The constants can be solved (see Exercise 4) to give the subsequent motions of the pendulums to be

$$x_A(t) = A cos\left(\frac{\omega_2 - \omega_1}{2} t\right) cos\left(\frac{\omega_2 + \omega_1}{2} t\right)$$

$$x_B(t) = A sin\left(\frac{\omega_2 - \omega_1}{2} t\right) sin\left(\frac{\omega_2 + \omega_1}{2} t\right) \tag{12}$$

The motions of the pendulums A and B exhibit a typical beat phenomenon. The motion can be understood as oscillations with a time period $4\pi/\left(\omega_2 + \omega_1\right)$ and a sinusoidally varying amplitude $A(t) = A cos(\frac{\omega_2 - \omega_1}{2} t)$ with the amplitude becoming zero with a period of $2\pi/\left(\omega_2 - \omega_1\right)$. As an example, Figure 3(a), 3(b) show plots of $x(t) = sin(2\pi t)sin(50\pi t)$ and $x(t)=cos(2\pi t)cos(50\pi t)$ vs. t respectively.

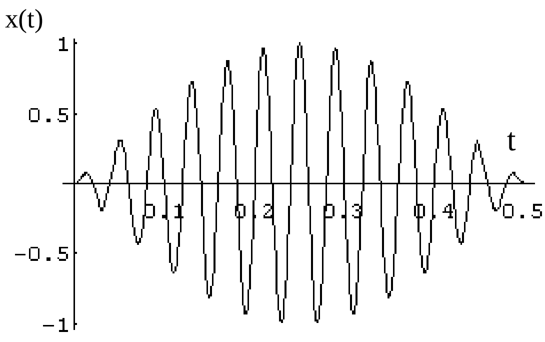


Fig. 3(a)

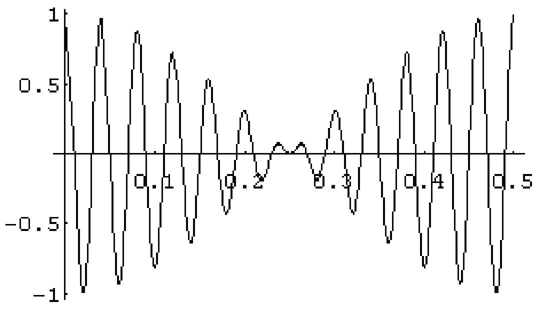


Fig. 3(b)

II. Set-up and Procedure

1. Uncouple the pendulums. Set small oscillations of both pendulums individually. Note the time for 20 oscillations and hence obtain the average time period for free oscillations of the pendulums and the natural frequency ω_0.
2. Couple the pendulums by hooking the spring at some position to the vertical rods of the pendulums. Ensure that the spring is horizontal and is neither extended nor hanging loose to begin with.
3. Switch on the bulb and observe the spot at the centre of the screen.
4. Excite the first normal mode by displacing both pendulums by the same amount in the same direction. Release both pendulums from rest. The spot on the screen should oscillate in the horizontal direction.
5. Note down the time for 20 oscillations and hence infer the time period T_1 and frequency ω_1 of the first normal mode.
6. With the spring at the same position excite the second normal mode of oscillation by displacing both pendulums in the opposite directions by the same amount and then releasing them from rest.
7. The spot on the screen should oscillate in the vertical direction. Note down the time for 20 oscillations and hence infer the time period T_2 and frequency ω_2.
8. Repeat these measurements for the spring hooked at 3 more positions on the vertical rods of the pendulums.

   (Part B)

9. For any one position of the spring (already chosen in Part A), now displace any one pendulum by a small amount and (with the other pendulum at its equilibrium position) release it from rest. Observe the subsequent motion of the pendulums. Try to qualitatively correlate the motion with the graph shown in Fig. 3. Measure the time period T of ***individual*** oscillations of the pendulum A and also the time period ΔT between the times when A comes to a total stop. Repeat these measurements three times for accuracy. Infer the time periods T_1 and T_2 of the normal modes from T and ΔT and compare with earlier results.

(Note: Your measurements will be more accurate only if you choose ℓ somewhat smaller than the total length L, i.e. choose a position of the coupling spring which is intermediate in position).

III. Exercises and Viva Questions

1. What are the normal mode oscillations of a system? How many normal modes will a system posses?
2. Infer the normal mode frequencies for the coupled pendulum by directly considering the motion in the two modes as shown in Figure 2.
3. Qualitatively explain why the first normal mode frequency is independent of the position of the spring while the second normal mode frequency increases with ℓ, the distance of the spring from the point of support.

4. For the case where pendulum A is lifted and released from rest derive the unknown constants A_1, A_2, φ_1, φ_2 in equation (11) to obtain the solution equation (12).
5. Explain the effect of damping on the motion. Redraw Figure 3 qualitatively if damping is present.
6. List all the approximations made in the theory of the double pendulum treated in the theory as against the actual apparatus used and estimate the error introduced. Also, consider possible sources of random errors while conducting the experiment.
7. Explain why the spot on the screen moves the way it does, i.e., horizontally when the 1st normal mode is excited and vertically when the 2nd normal mode is excited.
8. Describe and explain the motion of the spot on the screen when only one pendulum is displaced.
9. In Part B, derive the expressions for the normal modes ω_1 and ω_2 from the T and ΔT. What is the reason that the procedure asks you to choose a value of ℓ small compared to L for better accuracy?
10. Give some more examples of coupled oscillations from Physics or Engineering systems.

References:

1. "Vibrations and Waves", A.P. French, Arnold-Heinemann, New Delhi, 1972.
2. "The elements of Physics", I.S. Grant and W.R. Phillips, Oxford University Press, Oxford, 2001.

Experiment 2 (Part-I)

Velocity of Sound

Apparatus:

Audio frequency generator, speaker, microphone, Cathode Ray Oscilloscope (CRO), meter scale, large board (or wall), Thermometer (0-100°C)

Purpose of experiment:

i. To determine the velocity of sound

ii. To understand the operation of a CRO

Basic methodology:

Sound waves produced by an audio frequency generator are made to reflect off a large reflecting board forming standing waves. A microphone connected to a CRO serves to measure the amplitude of the sound. The wavelength of sound waves is obtained from the positions of the nodes.

I Introduction :

I.1 Standing waves are produced when two progressive sinusoidal waves of the same amplitude and wavelength interfere with each other. Consider two traveling waves traveling along the positive and negative *x* directions respectively

$$y_1(x,t) = A \sin (kx - \omega t) \quad (1)$$

$$y_2(x,t) = A \sin (kx + \omega t), \quad (2)$$

where *A* is the amplitude of the waves, $k = 2\pi/\lambda$ is the wave number and $\omega = 2\pi f$ is the angular frequency. The quantity $y(x,t)$ is the displacement of the medium at the point x and time t. When the two waves are made to interfere then by the principle of superposition, the net displacement is the sum of the individual displacements. Thus,

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\sin(kx-\omega t) + A\sin(kx+\omega t)$$

$$\Rightarrow \quad y(x,t) = (2A\sin kx) \cos\omega t \quad (3)$$

The resulting displacement, eq. (3), represents a wave of frequency ω, and an amplitude, $2A\sin kx$, which varies with the position *x*. The amplitude is zero for values of *kx* that give $\sin kx = 0$. These values are

$$kx = n\pi \quad for\ n = 0, 1, 2 \ldots.$$

Now $k = 2\pi/\lambda$. Therefore,

$$x = \frac{n\lambda}{2},\ n = 0, 1, 2\ldots \quad (4)$$

represent the positions of zero amplitude. These points are called *nodes*. Note that adjacent nodes are separated by $\frac{\lambda}{2}$, half a wavelength.

The amplitude of the standing wave has a maximum value of 2*A*, which occurs for values of *kx* that give $|\sin kx|$ = 1. These values are

$$kx = (n + \frac{1}{2})\pi,\ n = 0,\ 1,\ 2\ldots \tag{5}$$

The positions of maximum amplitude are called *antinodes* of the standing wave. The antinodes are separated by $\frac{\lambda}{2}$ and are located half way between pairs of nodes.

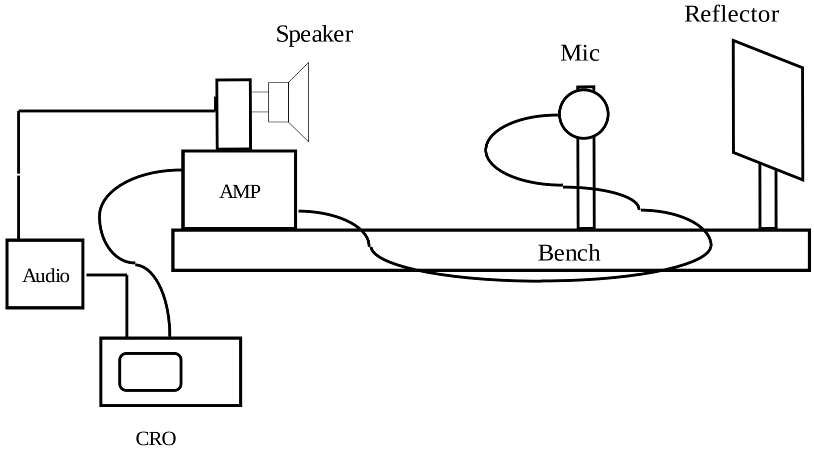
Now, a sound wave is a longitudinal wave representing displacement of particles in the medium and the resulting pressure variations. Traveling sound waves can also be taken to be represented by eqs. (1) and (2) with $y(x,t)$ denoting the longitudinal displacements of air particles. The velocity of the sound wave will be given by

$$v = \omega/k = f\lambda \tag{6}$$

In this experiment standing waves of sound are formed in air. The distance between successive nodes or antinodes is $\lambda/2$. By measuring the distance between the nodes, the wavelength can be determined and hence the velocity v of sound (knowing the frequency f). The apparatus of the experiment includes a cathode ray oscilloscope (CRO). One of the aims of the experiments is to provide a familiarity with the use of a CRO. The main features and controls of a CRO are described in the appendix to this experiment.

II. Set-up procedure:

Fig. 1 shows the basic set up of the apparatus.



PART A

Connect the audio frequency generator to the loudspeaker and adjust the controls of the generator so that the speaker produces a sound signal in the frequency range 1-10 kHz. Place the loud speaker facing a large board *B* (or a wall) at a distance of about one meter.

Connect the small microphone (mic) mounted on the bench. Connect the signal from the microphone to the *y*-channel of the CRT.

Select a proper scale for the horizontal time base to observe a stationary sinusoidal trace on the screen of the CRT. Adjust the vertical and horizontal positions so that the trace is symmetrically positioned on the screen. Observe that the amplitude of the trace changes as the position of the microphone is varied along the bench. Measure the period of the signal by reading the number of horizontal divisions separating the minima of the signal on the CRO screen. From the chosen scale for the time base determine the time interval between successive minima of the trace and hence calculate the frequency of the signal and compare with the frequency generated by the audio generator.

Next place the microphone close to the wall and move it away from the wall. Note down the positions of the nodes, i.e., where the amplitude of signal on the CRT screen becomes minimum. Note down the positions of at least five successive nodes. Repeat this measurement for three different frequencies.

PART B

Set the Trigger source for the oscilloscope to external Trigger. Connect the output of the speaker to the X input of the scope.

Observe the Lissajous pattern produced on the screen. Move the microphone along the bench and observe different Lissajous patterns. Sketch the observed patterns for the cases when the phase difference between X and Y signal are 0°, 90^o, 180°, 270^o.

(Note: due to small amplitude of the signals the observed pattern may be small in size. Also due to attenuation of the reflected signals there can be distortion of the pattern).

Select (say) the straight line pattern for 0^o or 180^o phase difference. Starting from the position nearest to the loud speaker move the microphone outwards towards the wall and note its positions for which the selected pattern on the screen repeats. The distance between two successive such positions corresponds to the wavelength of the sound wave.

Precautions:

1. *The microphone should be moved along the axis of the loudspeaker.*
2. *It is advisable to measure the position of nodes rather than antinodes*
3. *To lower error in the velocity determination the position of the nodes should be accurately determined.*
4. *The intensity of the CRO spot/pattern should be set LOW especially when the spot is stationary to avoid damage to the fluorescent screen.*
5. *Learn the functions of the various control knobs of the CRO before operating the CRO.*

III. Exercises and Viva Questions:

1. What is a traveling wave? Write down equations representing waves traveling along +ve and –ve x directions.
2. What is a standing wave? Explain by superposing appropriate traveling waves.
3. What are nodes and antinodes? Draw a rough diagram depicting the standing wave formed in the experiment. Is the point at the reflection board a node or an antinode?
4. On what factors does the velocity of sound depend? What is the effect of temperature, pressure and humidity on the velocity of sound?

5. What are Lissajous figures? Explain by construction how Lissajous patterns are produced when two perpendicular oscillations of phase differences 0°, 90°, 180°, 270° are superposed.

6. List the different sub-systems of a CRO and explain their operation and function.

7. Explain how a CRO can be used for voltage, frequency and phase measurements? How can a CRO be used for current measurement?

8. A periodic signal of 400 Hz is to be displayed so that 4 complete cycles appear on the oscilloscope screen, which has 10 horizontal divisions. To what settings should the Trigger source and sweep Time/div be set to allow this pattern to be displayed?

9. Explain what triggering, internal triggering and external triggering mean.

10. List the possible sources of error in this experiment and quantitatively estimate the error caused in the velocity measurement.

References:

1. "Fundamental of Physics", 6th ed., D. Halliday, R. Resnick and J. Walker, John Wiley and Sons Inc., New York, 2001.

2. "Physics", M. Alonso and E.J. Finn, Addison Wesley, 1992.

Appendix: Cathode Ray Oscilloscope

A cathode ray oscilloscope (CRO) is a convenient and versatile instrument to display and measure analog electrical signals. The basic unit of a CRO is a cathode ray tube (CRT). The CRO displays the signal as a voltage variation versus time graph on the CRT screen. By properly interpreting the characteristics of the display the CRT can also be used to indicate current, time, frequency and phase difference.

The basic subsystems of CRO are :

1. Display subsystem (CRT)
2. Vertical deflection subsystem
3. Horizontal deflection subsystem
4. Power supplies
5. Calibration circuits

Fig. 1 Show a schematic block diagram of how the subsystems are interconnected to produce the observed signal.

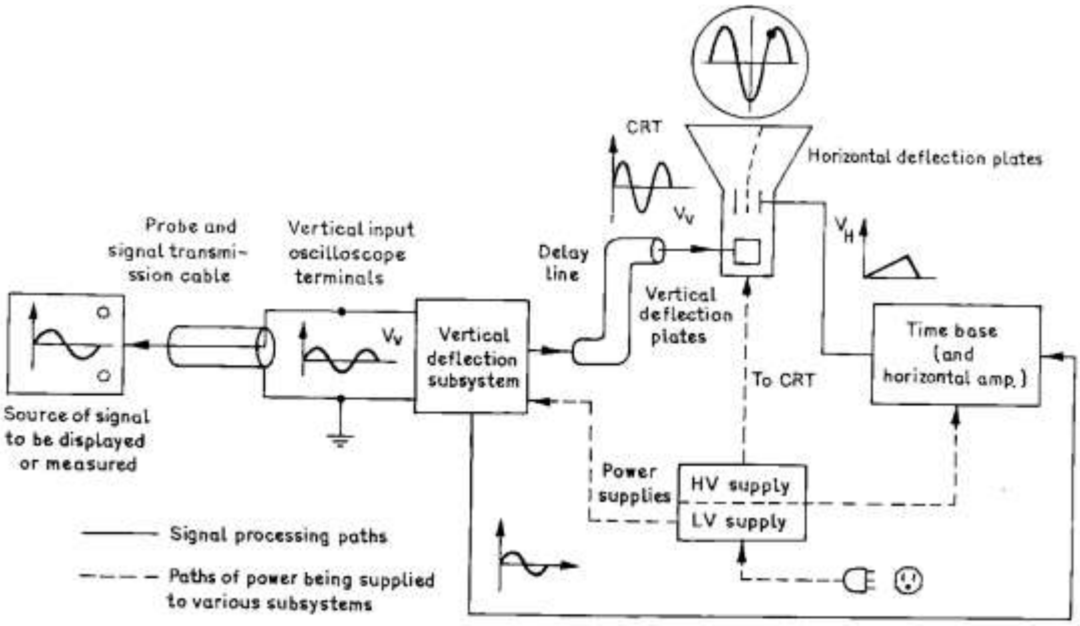


Fig. 2

The Signal is sensed at its source by the oscilloscope probe. The signal voltage is then transmitted to the oscilloscope along a coaxial cable and fed to the vertical display subsystem. After suitable amplification, the input signal is applied to the vertical deflection plates of the CRT. This causes the electrons emitted by the electron gun in the CRT to deflect vertically in proportion to the amplitude of input voltage.

There is a simultaneous deflection of the electron produced by the Horizontal deflection subsystem. The amplified input signal is also fed to the horizontal deflection subsystem. In order to produce a Y-t display a voltage that causes the horizontal position of the beam to be proportional to time must be applied to the

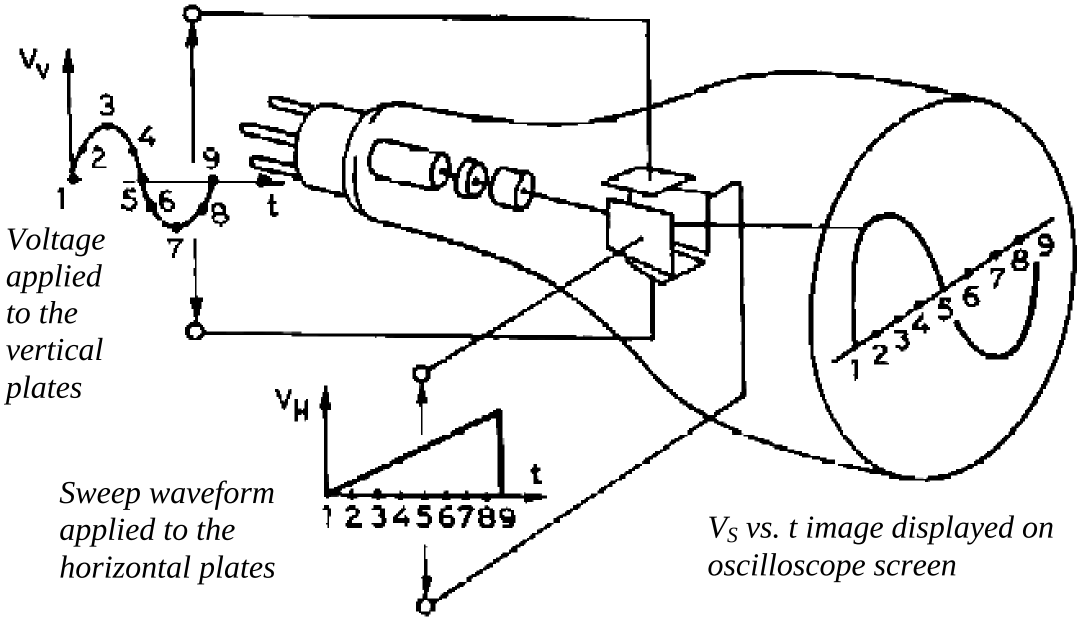


Fig. 3

horizontal deflector plates. The horizontal voltage called the sweep waveform is generally of a sawtooth form. Fig. 3 shows has the time variation of the input signal is displayed with help of the sweep waveform.

The synchronization of the input signal with the sweep wave form is carried out by the time base circuitry. A triggering signal (which could be the input signal or an external signal) is fed to a pulse generator. The emitted pulses are fed to a sweep generator which produces a series of sweep wave forms.

Controls on the CRO front panel:

A. General

Intensity: Controls the intensity of the spot on the screen by controlling the number of electrons allowed to pass to the screen

Focus: ***Controls the focussing of the electron beam.***

X Shift/Y Shift: ***Changes the deflection voltages by a constant amount to shift the signal vertically or horizontally.***

B. Vertical Deflections Subsystem

Vertical sensitivity: Amplifier for the vertical deflection subsystem calibrated in terms of sensitivity. The input voltage can be determined from the deflection of the signal. For eg., if the vertical sensitivity is set at 50 mV/div and the vertical deflection is 4 div, then the input voltage is $50 \times 4 = 200$ mV.

Var (V/dir): Vernier control for continuous vertical sensitivity.

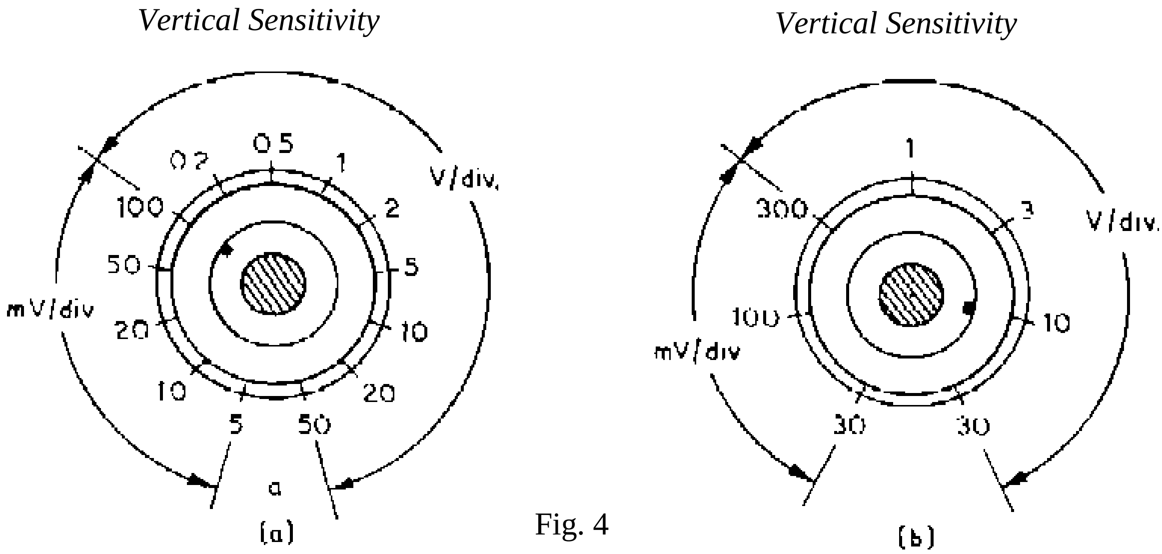


Fig. 4

C. Horizontal Deflection Subsystem

Sweep Time (Time/dir) : Controls the sweep time for the spot to move horizontally across one division of the screen when the triggered sweep mode is used.

Var (Time/div) : Vernier control for continuous change of sweep time.

Trigger : Selects the source of the trigger signal which produces the sweep waveform.

Internal Trigger : The output of the vertical amplifier is used to trigger the sweep waveform.

External Trigger : An external signal must be applied to the X inputs to trigger the sweep wave form.

Sweep Magnifier : Decreases the time per division of the sweep waveform.

Trigger Level : Selects amplitude point on trigger signal that causes sweep to start.

Trigger Mode : AUTO provides normal triggering and provides baseline in absence of trigger signal. NORM permits normal triggering but no sweep in absence of triggering. TV provides triggering on TV field or TV line.

References:

1. “Student Reference Manual for Electronic Instrumentation”, S. E. Wolf and R.F.M. Smith, PHI, New Delhi,1990.
2. “Basic Electronic Instrument Handbook”, C. F. Coombs, McGraw Hill Book Co., 1972.

Experiment 2 (Part-II)

The vibrating string

Apparatus:

Tuning Fork, Electrically Driven Oscillator, String, Peg, Weights, DC Power Supply

Purpose of Experiment:

i) To study normal modes of transverse vibration of a stretched string
ii) To determine the frequency of vibration of the tuning fork driving the string

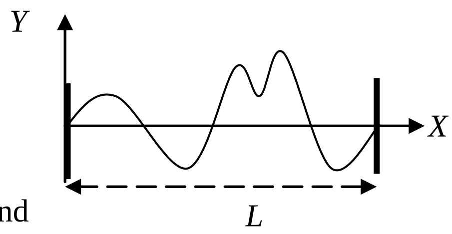
Basic Methodology:

A string is stretched horizontally and one end of it is set into transverse vibration of a fixed frequency. By continuously changing the length of the string, one observes the different normal modes of vibration. Moreover, by using the measured values of the length of the string, its mass per unit length and the tension in it at a particular normal mode vibration, one can determine the frequency of the oscillator driving the string.

I Introduction:

I.1 When a string of length L is stretched and set into a transverse vibration with both ends fixed, its transverse displacement $y(x,t)$ is governed by the equation

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$



where $v = \sqrt{\frac{T}{\mu}}$, T is the tension in the string and μ is its mass per unit length. Here, the x axis is assumed to be along the string and y axis along the direction of transverse vibration (See Fig.1).

Fig. 1

I.2 A general vibration of the string is any solution $y(x,t)$ of eq. (1) with the boundary conditions $y(0,t) = y(L,t) = 0$ for all t. Such a general vibration (See Fig. 1) is not characterized by any definite frequency of vibration.

I.3 There are, however, particular modes of vibration in which all points on the string vibrate in a simple harmonic motion with a common frequency ν but with continuously varying amplitudes. These modes of vibration are known as normal modes and the corresponding common frequencies, the normal frequencies. Any general vibration of the string is a superposition of these normal modes with appropriate amplitudes. These normal modes are given as :

$$y_n(x,t) = A\sin\left(\frac{n\pi x}{L}\right)\cos(2\pi\nu_n t) \quad \text{for } n = 1,2,3, \ldots\ldots\ldots\ldots$$

where, ν_n s, the normal frequencies, are given as

$$\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \ , \qquad \text{for } n = 1,2,3 \qquad (2)$$

(Check that the $y_n(x,t)$ s are solutions of eq. (1)). A normal mode for a given n above is called the n^{th} normal mode or $(n-1)^{th}$ harmonic. The mode $n = 1$ is called the fundamental mode. Clearly, in a normal mode, a point x on the string vibrates with a frequency ν_n and amplitude $A\sin\left(\frac{n\pi x}{L}\right)$ which varies from point to point on the string. (See Fig. 2).

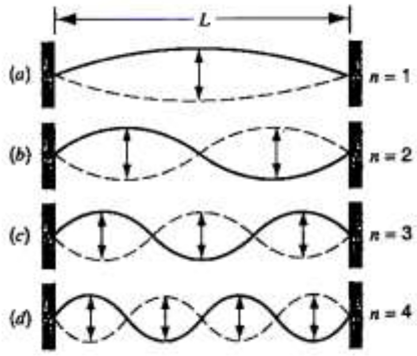


Fig.2. First four normal modes (from (a)-(d)) of stretched string of length L with fixed end points

I.4 From the expression for ν_n s above, we see that for a given length L and a tension T, the string can have normal frequencies that are integral multiples of $\frac{\pi}{L}\sqrt{\frac{T}{\mu}}$. Conversely, as is the case in the present experiment, if one end of a string is driven transversely at a fixed frequency ν and negligible amplitude (amplitude should be negligible so that the condition of fixed end points holds), then it can vibrate in one of its normal modes provided L and T are such that the condition

$$\frac{n\pi}{L}\sqrt{\frac{T}{\mu}} = \nu \qquad (3)$$

is satisfied for some + ve integer n (mode of vibration). In this case, it is the frequency of the mode which is fixed and the mode is selected by appropriate values of L & T. In other words, when L & T are such that one of the normal frequencies matches the driving frequency, the string resonates with that frequency.

I.5 Suppose a particular mode, say n^{th} , is obtained for a certain combination of L and T values. We then have ν given by eq. (3). One can thus measure the value of the driving frequency.

II Set-up and Procedure :

1. Clamp the pulley with its support to the edge of a table. Place the stand of the tuning fork about 50 cm away from the pulley on the table and clamp the tuning fork to it in a horizontal fashion so that it faces the pulley. Secure the electrically driven oscillator to the tuning fork (not shown in the figure) and connect it to the DC power supply (Battery eliminator). Tie one end of the string to the hook attached to the tuning fork and pass the other end over the pulley. Tie the peg to this end so that the string is taut and the free end with the peg hangs vertically down. The string, before it passes over

the pulley, should lightly touch a small groove cut into the edge of a vertical plate placed in front of the pulley. This point is one of the fixed ends of the string, the other being the point where it is tied to the tuning fork. See Fig. 3 below for the above set-up.

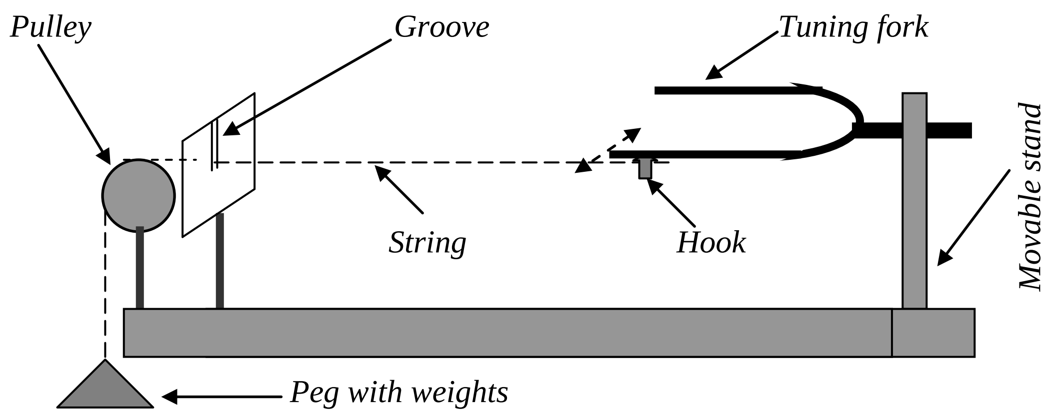


Fig. 3

2. Place a weight of 40 gm on the peg. The peg itself weighs 10 gm, so that the net tension in the string is 0.5 N. This is the tension for which the fundamental mode occurs for a length of the string that is about 20 cm and one can, therefore, observe few more harmonics within a total length of about 1.5 m.

3. Switch on the DC power supply so that the oscillator and consequently the tuning fork start vibrating. The oscillator is used to maintain the vibration of the tuning fork, without which it would stop quickly due to damping.

4. Move the stand holding the tuning fork slowly towards the pulley. When the length of the string is about 20 cm, you will see it vibrate in its fundamental mode. Make finer adjustment of the length so that the amplitude of vibration at the middle (anti node) is as large as possible and the two ends of the string, one near the pulley and other at the hook attached to the tuning fork are perfect nodes (zero displacement).

5. Switch off the power supply. When the string comes to rest, measure its length using a meter scale.

6. Switch on the power supply again. Move the tuning fork slowly away from the pulley until you see the string vibrate in its first harmonic. This will occur at twice the length for the fundamental mode. Do finer adjustment of the length so that the middle as well as the two ends of the string are perfect nodes. Switch off the power supply and measure the length of the string. Repeat the above to obtain the next two modes and measure the corresponding lengths of the string.

7. Increase the tension in the string by adding more weights to the peg and do the same as for the first value of the tension. Repeat for one more value of the tension.

Precautions:

1. The vibrating part of the string should be perfectly horizontal

2. The vertical plane containing the vibrating part of the string should be the same as the plane of the pulley, i.e, the string should not pull at the pulley sideways.

3. The peg carrying the weights should be stationary and it should not touch the ground.

III Exercises and Viva Questions:

1. What is a normal mode of a vibrating system? How many normal modes does a stretched string of length *L* have?
2. If a stretched string is plucked at a point and left to vibrate, will it vibrate in a normal mode? If not, how is this vibration related to the normal mode vibrations?
4. The tuning fork used in the experiment is a heavy one. Is there any particular reason to have such a heavy tuning fork ?
5. What is the purpose of having the extra oscillator (the one driven by the DC power supply)?
6. Suppose the string is vibrating in its fundamental mode. If the tension in the string is reduced, in order to obtain the fundamental mode for the new tension, will you move the tuning fork towards or away from the pulley?
7. The tuning fork and the electrically driven oscillator have different natural frequencies. Which of these two frequencies goes into the calculations in the experiment?
8. For two different values of the tension, T_1 and T_2 respectively, with $T_1 < T_2$, which one will produce more number of normal modes for the given total length of the string?
9. What are the assumptions under which the equation of the vibrating string as given in eq. (1) holds?
10. If the string, instead of being held horizontally, is held vertically and set into transverse vibration, will its normal modes be different?

References:

1. "Waves and Vibrations", A.P. French, Arnold-Heinemann, New Delhi, 1972

2. "Physics", R. Resnick, D. Halliday and K.S. Krane, Vol. 1, 5th Ed., John Wiley & Sons, Singapore, 2002

Experiment 3

Resonance in LCR circuit

Apparatus:

Oscillator (1 to 150 kHz), variable inductor, variable capacitor, resistance box, *AC* mill-voltmeter.

Purpose of experiment:

To study resonance effect in series and parallel *LCR* circuits.

Basic methodology:

In the series *LCR* circuit, an inductor (*L*), a capacitor (*C*) and resistance (*R*) are connected in series with a variable frequency sinusoidal emf source and the voltage across the resistance is measured. As the frequency is varied, the current in the circuit (and hence the voltage across *R*) becomes maximum at the resonance frequency $\nu_0 = \dfrac{1}{2\pi\sqrt{LC}}$. In the parallel *LCR* circuit there is a minimum of the current at the resonance frequency.

I Introduction :

I.1 There is in general an analogy between resonating mechanical systems (like a driven spring mass system) and electrical systems involving inductors, resistor and capacitors. In the electrical case it is the charge *q(t)* on the capacitor (or the current *I=dq/dt*) that satisfies a differential equation analogous to the displacement of the mass in the familiar spring mass system.

Consider the circuit Fig.1, consisting of an inductor (*L*), capacitor (*C*) and resistance (*R*) connected in series with a source of sinusoidally varying emf $\varepsilon(t) = \varepsilon_0 \cos \omega t$. Equating the voltage drops across the resistor and capacitor to the total emf, we get,

$$RI + \frac{q}{C} = V_L + \varepsilon_0 \cos \omega t$$

$$= -L\frac{dI}{dt} + \varepsilon_0 \cos \omega t. \qquad (1)$$

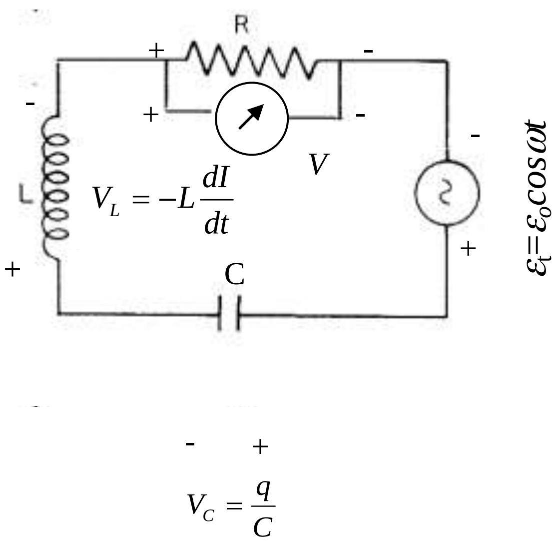


Fig. 1

Differentiating the equation with respect to time and rearranging, we get

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} = -\omega\varepsilon_0 \sin \omega t, \qquad (2)$$

which is analogous to the equation of motion for a forced damped oscillator.

The current $I(t)$ has the solution .

$$I(t) = I_0 \cos(\omega t - \alpha), \tag{3}$$

where, I_o exhibits resonance behavior. The amplitude I_0 is given by

$$I_0 = \frac{\varepsilon_0}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}} \tag{4}$$

and

$$\tan\alpha = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{5}$$

gives the phase of the current relative to applied emf. We can write $I_0 = \frac{\varepsilon_0}{Z}$, where

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} \tag{6}$$

is the impedance of the circuit. The reactance X of the circuit is

$$X = \omega L - \frac{1}{\omega C} \tag{7}$$

so that the impedance Z is given by

$$Z = \left(R^2 + X^2\right)^{1/2}.$$

Clearly the impedance will be minimum (and I_0 will be maximum) at resonance condition when the reactance vanishes, i.e., at the angular frequency (known as resonance frequency),

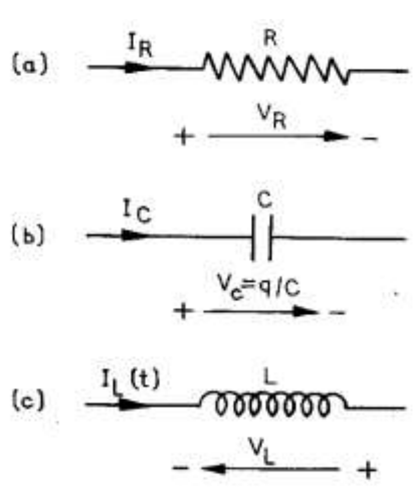
$$\omega_O = \frac{1}{\sqrt{LC}} \tag{8}$$

which is the natural frequency of electromagnetic oscillations in *LCR* circuit without an external source of emf.

I.2 Resistance, Capacitance and Inductance in *AC* circuits: Consider a resistor with a voltage drop $V_R = V_{R0}\cos\omega t$ across it (Fig. 2a). By Ohm's law the current through the resistor is

$$I_R = \frac{V_R}{R} = \frac{V_{R0}}{R}\cos\omega t \tag{9}$$

The current and voltage across a resistor are in phase.



In the case of a capacitor (Fig. 2b) the current $I_C = \frac{dQ}{dt}$ where q is the charge on the capacitor. If the potential drop across the capacitor is $V_C = V_{C0} cos\omega t$, the charge

$q = CV_C = CV_{C0} cos\omega t$.

Fig. 2

Thus, $I_C = -\omega CV_{C0} sin\omega t = \omega CV_{C0} cos\left(\omega t + \frac{\pi}{2}\right)$

(10)

Thus, the current through the capacitor is ahead of the voltage by phase angle $\pi/2$, Consider now an inductor (Fig. 2c) with current $I_L(t) = I_{L0} cos\omega t$. Assume that the current flows and increases in the direction shown. The back emf induced in the inductor opposes the current and the potential drop across the inductor is

$$V_L = L\frac{dI}{dt} = -\omega L\, I_{L0}\, sin\omega t = \omega L\, I_{L0}\, cos\left(\omega t + \frac{\pi}{2}\right) \quad (11)$$

The voltage across the inductor is ahead of the current in phase by an angle $\pi/2$.

I.3 Complex Impedance:

It is convenient to use complex phasors to represent the current and voltage in an *AC* circuit. For example, the phasor $\overline{V} = V_0 e^{j\omega t} = V_o(cos\omega t + j\,sin\omega t)$ represents a sinusoidally varying voltage $V_0 cos\omega t$ which is its real part. For any component *A* we define its complex impedance by $\overline{V}_A = \overline{Z}_A \overline{I}_A$. We write

$$\overline{Z} = R + jX ,$$

where the real part of $\overline{Z}$ is the resistive impedance (R), while the imaginary part of $\overline{Z}$ is the reactive impedance (X).

The complex impedances of the resistor, capacitor and inductor can be obtained by generalizing eqs. (9), (10) & (11) to phasor equations :

$$\overline{I}_R = \frac{1}{R}\overline{V}_R \Rightarrow \overline{Z}_R = R \quad (12)$$

$$\overline{I}_C = \omega CV_{C0}\, e^{j\left(\omega t + \frac{\pi}{2}\right)} = j\omega C\, \overline{V}_C \Rightarrow \overline{Z}_C \Rightarrow \overline{Z}_C = \frac{1}{j\omega C} \quad (13)$$

$$\overline{V}_L = \omega LI_{L0} e^{j\left(\omega t + \frac{\pi}{2}\right)} = +\,j\omega LI_L \Rightarrow \overline{Z}_L = j\omega L \quad (14)$$

Thus, the impedance of a resistor is its resistance itself, while the impedance of a capacitor and inductance are reactive with $X_C = -1/(\omega C)$ and $X_L = \omega L$.

It can be shown from Kirchhoff's rules that complex impedances in series or parallel combine just like resistors in series or parallel. Thus, for the series *LCR* circuit Fig 1, the net impedance of the circuit is

$$\overline{Z} = \overline{Z}_R + \overline{Z}_C + \overline{Z}_L = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

(15)

The current in the circuit is then

$$\bar{I} = \frac{\bar{\varepsilon}}{\bar{Z}} = \frac{\varepsilon_0 e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = I_0 e^{j(\omega t - \alpha)} \tag{16}$$

From eq. (16) it can be easily seen that

$$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \left(= Re\,\bar{I}\right)$$

and

$$\alpha = tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

which clearly reproduce eqs. (4) and (5). The physical current in the circuit is, of course, the real part of the phasor $\bar{I}$ in eq (16).

I.4 Parallel *LCR* circuit:

Consider now the parallel *LCR* circuit shown in Fig. 3. The current through the

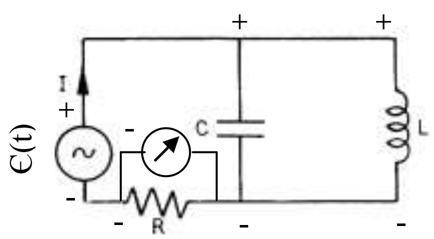


Fig. 3

resistor can be found by calculating the equivalent impedance of the circuit

$$\bar{Z} = \bar{Z}_R + \frac{1}{\frac{1}{\bar{Z}_C} + \frac{1}{\bar{Z}_L}} = R + \frac{\bar{Z}_L \bar{Z}_C}{\bar{Z}_C + \bar{Z}_L} = R - j\frac{L/C}{\omega L - \frac{1}{\omega C}} \tag{17}$$

Thus

$$\bar{I} = \frac{\bar{\varepsilon}}{\bar{Z}} = \frac{\varepsilon_0 e^{j\omega t}}{R - j\left(\frac{L/C}{\omega L - \frac{1}{\omega C}}\right)} = I_0\, e^{j(\omega t + \phi)} \tag{18}$$

The magnitude of the current I_0 is given by

$$I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\dfrac{L/C}{\omega L - \dfrac{1}{\omega C}}\right)^2}} \tag{19}$$

Viewed as a function of ω, it is clear that I_0 is now a minimum (the impedance in the denominator is maximum) when $\omega L = \frac{1}{\omega C}$, or, where

$$\omega = \frac{1}{\sqrt{LC}} = \omega_o \tag{20}$$

and is known as "resonance frequency" even through it corresponds to an amplitude minimum.

(Note: The amplitude of the current in eq. (19) strictly falls to zero at $\omega = \frac{1}{\sqrt{LC}}$ since the denominator tends to infinity. This is because we have considered idealized (i.e. resistance-less) capacitor and inductor. A finite value of the current amplitude at resonance will be obtained if resistive impedance is included for these components).

I.5 Power Resonance:

The power dissipated at the resistor is $P = IV = I^2R = V^2/R$. From eq. (3) for the series resonance circuit, the power dissipated at the resistor is

$$P = I_o^2 R\cos^2(\omega t - \alpha), \tag{21}$$

where I_0 is given by eq. (4). The average power dissipated over one cycle is

$$\bar{P} = \frac{I_0^2 R}{2} = \frac{\varepsilon_0^2 R}{2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} \tag{22}$$

Fig. 4 shows graph of $\bar{P}$ as a function of the driving frequency ω. The maximum power value, $\bar{P}_m$, occurs at the resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. It can be shown that to a good approximation, that the power falls to half the maximum value, $\bar{P}_m/2$ at $\omega = \omega_0 \pm \frac{\gamma}{2}$. Here γ is related to damping in the electrical circuit and is given by $\gamma = R/L$.

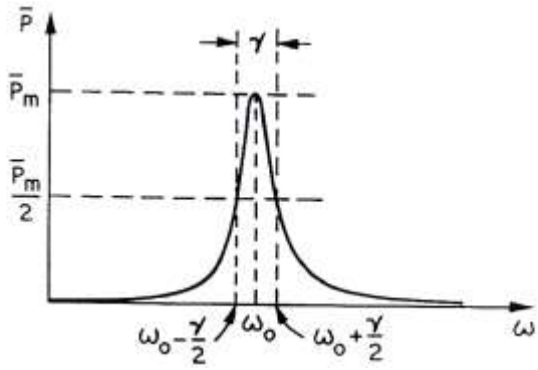


Fig. 4

The width or range of ω over which the value of $\overline{P}$ falls to half the maximum value at resonance is called the Full Width at Half Maximum (*FWHM*). The *FWHM* is a characteristic of the power resonance curve and is related to the amount of damping in the system. Clearly $FWHM = \gamma = \frac{R}{L}$. One also define the quality factor Q as $Q = \frac{\omega_0}{\gamma} = \frac{1}{R}\sqrt{\frac{L}{C}}$ which is also a measure of damping. Large Q (small R) implies small damping while small Q (large R) implies large damping.

Clearly we have, $$FWHM = \gamma = \frac{\omega_0}{Q} \qquad (23)$$

Thus, the quality factor Q can be determined from the *FWHM* of the power resonance graph.

II. Set-up and Procedure

1. The series and parallel *LCR* circuits are to be connected as shown in Fig.1 and Fig 3.
2. Set the inductance of the variable inductance value and the capacitance of the variable capacitor to low values (L ~ 0.01H, C ~ 0.1μF) so that the resonant frequency $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$ is of the order of a few kHz.
3. Choose the scale of the *AC* millivoltmeter so that the expected resonance occurs at approximately the middle of that scale.
4. Vary the frequency of the oscillator and record the voltage across the resistor.
5. Repeat (for both series and parallel *LCR* circuits) for three values of the resistor.

III. Exercises and Viva Questions.

1. Write down the Newton's law for a forced damped harmonic oscillator and map the electrical quantities appearing in eq. (2) with the corresponding mechanical quantities.
2. Verify that the solution, eq. (3) satisfies the differential equation (2).
3. Distinguish between resistive impedance and reactive impedance. What is the effect of a reactive impedance on the current and voltage in an *AC* circuit? In a *DC* circuit?
4. For the circuit shown with emf $\varepsilon(t) = \varepsilon_0 \cos\omega t$, determine the current $I(t) = I_0 \cos(\omega t - \alpha)$. (i.e. determine the amplitude I_0 and phase α)

5. Calculate the (resistive or reactive) impedance of the components L, C, and R at resonance for series and parallel circuits, for your experiment.

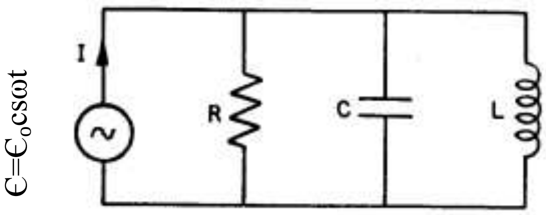


Fig. 5

6. Why does the series circuit give a power maximum at resonance while the parallel circuit lead to a power minimum?

7. The *AC* millivoltmeter gives the 'rms' value of the voltage across the resistor, i.e V_{rms}. If $V = V_0 \cos\omega t$, what is V_{rms}? Show that the average power $\overline{P} = V_{rms}^{\ 2} / R$.

8. Show that eq. (22) can be rewritten as

$$\overline{P} = \frac{\varepsilon_0^{\ 2} R L}{2C} \frac{1}{\left[Q^2 + \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^2 \right]}.$$

9. Qualitatively plot the power resonance curve for increasing values of Q. Show that the *FWHM* of the power resonance curve is approximately given by is $\gamma = \frac{\omega_0}{Q}$.

10. Argue why the power maximum (minimum) for the series (parallel) *LCR* circuit increases (decreases) with increasing R.

References:

1. “Physics”, M. Alonso and E.J. Finn, Addisson-Wesley, 1992.
2. “Linear Circuits”, M. E. Van Valkenburg and B.K. Kinariwala, Printice Hall, Englewood Cliffs, NJ, 1982.

<u>Experiment 4</u>
Electromagnetic Induction

Apparatus:
Metallic semi-circular arc (radius 40 cm), supporting frame, movable weights, bar magnets, measurement board consisting of voltmeter, milli-ammeter, resistance, condenser and diode.

Purpose of experiment:

To verify Faraday's law of electromagnetic induction.

Basic methodology :
A bar magnet is made to pass through a coil. The resulting emf produced by Faraday's effect charges a capacitor. The voltage of the capacitor is a measure of the induced emf.

I. Introduction

I.1 Faraday's law states that a change in magnetic flux (Φ) through a closed conducting circuit induces an electro motive force (emf) ε in the circuit:

$$\varepsilon = -\frac{d\Phi}{dt} \qquad (1)$$

The emf ε is proportional to the rate of change of the flux through the coil. The minus sign is related to the fact that the induced emf opposes the change in the flux linking the circuit. In *MKS* unit ε has units of Volts (*V*), while Φ has units of Weber (*W*). In this experiment we will measure certain effects leading from Faraday's law and will hence indirectly verify the law.

The setup (Fig. 1) basically consists of a bar magnet attached to a metallic arc. The frame of the arc is suspended at the center so that the whole frame can freely oscillate in its plane. Movable weights are provided on the diagonal arm whose position can be altered, leading to a variation of the period of oscillation from about 1.5 sec. to 3 sec.

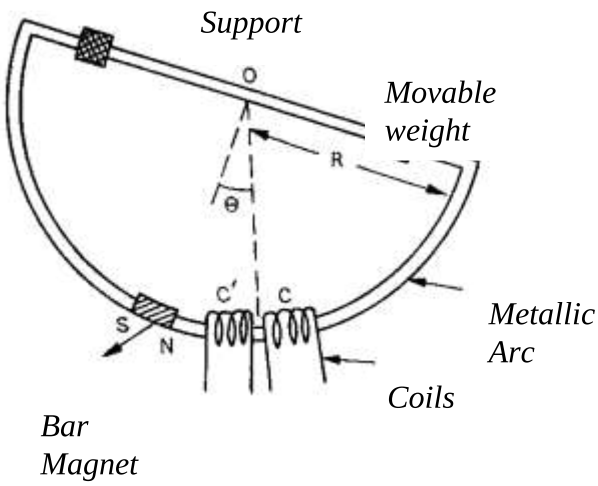


Fig. 1

As it oscillates the magnet passes through two copper coils (connected in series) of about 10,000 turns.

As the magnet passes through the coils the flux Φ through the coils changes with time as shown in Fig. 2. The induced emf ε is generated in the coils in the form of two pulses with opposite signs for each swing. The pulse width τ is the time over which the flux through the coil changes during a swing. The maximum value ε_0 of the emf corresponds to the maximum value of $\left|\frac{d\Phi}{dt}\right|$. This is related to the maximum velocity v_{max} of the magnet.

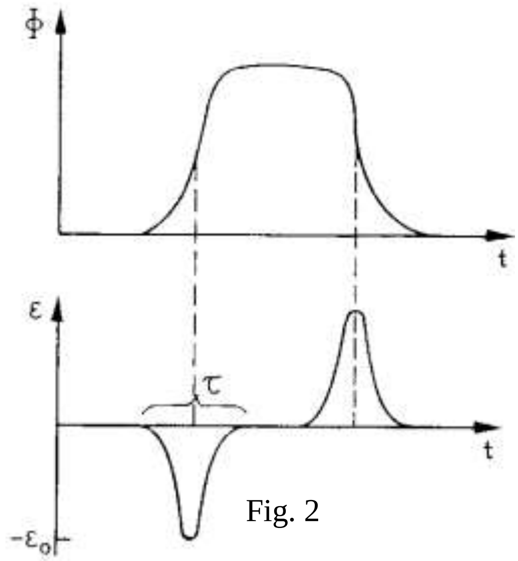


Fig. 2

I.2 *Calculation of* v_{max}

The maximum velocity of the magnet is clearly obtained at the equilibrium point i.e. at the bottom of the swing. The velocity v_{max} *can be easily calculated. If* M *is the mass of the frame + magnet and* l *is the distance of the center of mass from the point of suspension of the frame + magnet and* θ_0 *is the initial release angle, then by conservation of energy we have*

$$\tfrac{1}{2} I\omega_{max}^2 = Mgl\,(1-\cos\theta_0) = 2\,Mgl\,\sin^2\frac{\theta_0}{2} \qquad (1)$$

Thus

$$\omega_{max} = 2\sqrt{\frac{Mgl}{\mathrm{I}}}\,\sin\frac{\theta_0}{2} \qquad (2)$$

The quantity $\sqrt{\frac{Mgl}{\mathrm{I}}}$ is actually the natural frequency of small oscillations. Thus if T is the period of small oscillations then

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{M}gl}} \qquad (3)$$

Using eq(3) and $v_{max} = R\omega_{max}$ (R is the radius of the arc) we get

$$v_{max} = \frac{4\pi R}{T}\,\sin\frac{\theta_0}{2}. \qquad (4)$$

I.3 *We now gives a rough argument that the maximum value of the emf* $\varepsilon_{max} = \varepsilon_0$ *is proportional to* v_{max}.

As the magnet moves, the flux Φ through the coil changes. Clearly $\Phi = \Phi(\theta)$ where θ is the angular position of the magnet. Hence

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d\Phi}{d\theta}\frac{d\theta}{dt} = -\frac{d\Phi}{d\theta}\omega \tag{5}$$

Thus, ε is also a function of θ, i.e.

$$\varepsilon(\theta) = -\frac{d\Phi(\theta)}{d\theta}\,\omega(\theta) \tag{6}$$

At the equilibrium point $\omega = \omega_{max}$ but $\frac{d\Phi}{d\theta} = 0$ hence $\varepsilon_{eq} = 0$. The maximum of the emf ε_{max} occurs at and angle θ_{max} slightly before the equilibrium point

$$\varepsilon_{max} = \varepsilon_0 = -\left.\frac{d\Phi}{d\theta}\right|_{\theta_{max}} \omega(\theta_{max}) \tag{7}$$

Since the point θ_{max} is close to the equilibrium $\theta = 0$, $\omega(\theta_{max}) \approx \omega_{max} = \frac{v_{max}}{R}$. Hence

$$\varepsilon_{max} = \varepsilon_0 \approx -\frac{1}{R}\left.\frac{d\Phi}{d\theta}\right|_{\theta_{max}} v_{max} \tag{8}$$

Thus $\varepsilon_0 \propto v_{max}$ approximately and the constant of proportionality depends only on the geometry of the apparatus and is independent of the release angle θ_0. Hence a graph of ε_0 *vs.* v_{max} is expected to approximate a straight line.

I.4 In this experiment we will measure ε_0 *by charging a capacitor by the induced emf. The capacitor is connected in series with the coil along with a diode and a resistance* R. *The resistance* R_{int} *is the internal resistance of the coil and forward resistance of diode and is about 500Ω. The diode allows current to flow only in one direction and hence the capacitor charges only during one swing of the complete oscillation. If the time constant* RC *is small compared to the pulse width* τ *then the capacitor gets fully charged to the maximum voltage* ε_0 *in the swing. However if* RC>τ *then the capacitor gets fully charged only after several swings.*

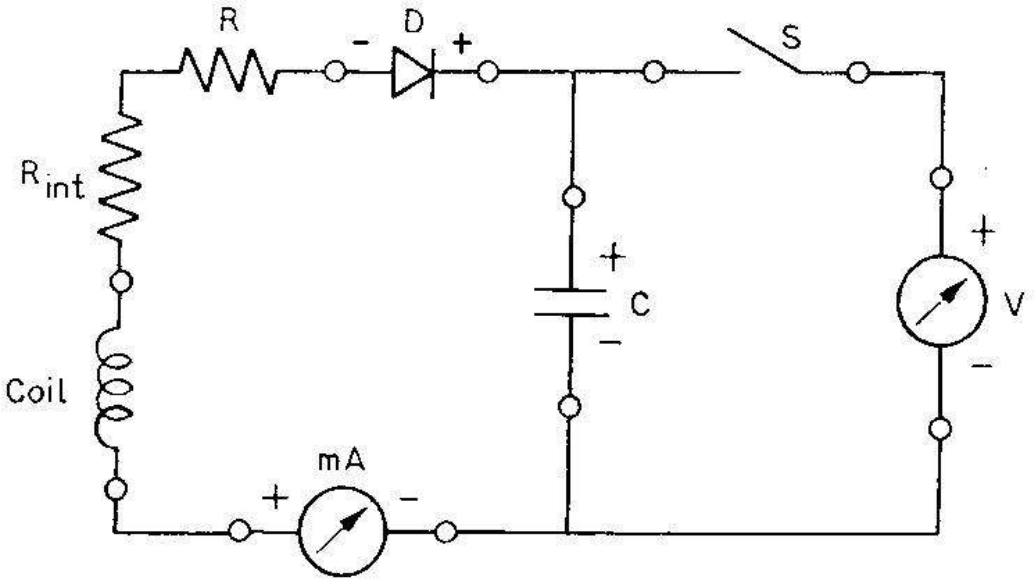


Fig. 3

The voltage across the capacitor after n *swings can be measured by closing the switch* S *and discharging the capacitor through a voltmeter.*
The total charge delivered to the capacitor during each swing is

$$q = \int_{initial}^{final} \frac{\varepsilon dt}{R} = -\int_{i}^{f} \frac{1}{R}\frac{d\Phi}{dt}\,dt$$

$$= \frac{1}{R}\left(\Phi_i - \Phi_f\right) = \frac{\Delta\Phi}{R} \qquad (9)$$

I.5 Electromagnetic damping in an oscillating system.

Successive oscillations of the metal arc do not have the same amplitude. This is due to damping whose primary sources are i) air friction ii) friction at point of suspension iii) electromagnetic damping to Lenz's law.

The damped oscillation of the system can be modeled by the differential equation (assuming small damping).

$$\frac{d^2\theta}{dt^2} + \left(\frac{\omega_0}{Q}\right)\frac{d\theta}{dt} + \omega^2\theta = 0 \qquad (10)$$

where $\omega_0^2 = \sqrt{\frac{Mgl}{I}}$. The third term arises from the restoring force while the second term proportional to $\frac{d\theta}{dt}$ represents damping. The strength of the damping is characterized by the parameter Q, called quality factor of the system. Small Q implies large damping while large values of Q (Q>1) represents small damping. The solution $\theta(t)$ to eq. (10) can be shown to be oscillatory but with an amplitude which decreases with time as

$$\theta_A(t) = \theta_{A0} e^{-\frac{\omega_0 t}{2Q}} \qquad (11)$$

Thus after n oscillations i.e., $t = nT = n\frac{2\pi}{\omega_0}$ the amplitude decreases from the initial amplitude θ_{A0} by

$$\theta_{An} = \theta_{A0}\, e^{-\frac{\pi n}{Q}} \qquad (12)$$

Thus, $$ln\theta_{An} = ln\theta_{A0} - \frac{\pi}{Q}n \qquad (13)$$

A plot of $ln\ \theta_{An}$ vs. n is expected to be a straight line. The quality factor can be read off from the slope.

II. Setup and Procedure

PART A : Measurement of time period T for small oscillations

1. Make sure that the equilibrium position of the metal arc + magnet is at $\theta = 0^0$. If not adjust the position of the weights to ensure this.
2. Check that the oscillations of the arc through the coils are free and that the arc does not touch the sides of the coil when oscillating.

3. Displace the metal arc by a small angle (5-10^0) and measure the time taken for a few (say 5) oscillations. The time period T can then be obtained.

   Repeat for 3 different displacement angles.

PART B : Measurement of ε_0

1. **Connect the circuit as in Fig. 3 Take C = 100 μF and R to be small (~ 100 Ω). Connect the two coils in series.**
2. Keep switch in the off position.
3. Choose an initial displacement θ_0 (say 40°) and release the magnet. As the induced current flows in the circuit the milliammeter registers kicks. The kicks stop after a few oscillations when the capacitor has become fully charged.
4. Flip the switch to the ON position and measure ε_0 as the maximum voltage recorded by the voltmeter.
5. Repeat for different values of θ_0 = 40°, 35°, 30°, 25°, 20°, 15°, 10°.
6. Calculate v_{max} for each case and plot ε_0 *vs.* v_{max}.

PART C : Charge delivered to the capacitor.

1. Choose a large value of R (say 1 kΩ) in the circuit of Fig. 3. The time constant RC is thus greater than τ (τ is approximately estimated by dividing the magnet length by v_{max}).
2. With a given release angle θ_0, measure the voltage V across the capacitor after n complete oscillation, n = 1,2,…,6.

   (Caution : Each time, i.e., after n oscillations, prevent further oscillations by stopping the frame by hand and measure V.

   Also make sure that the capacitor is completely discharged each time before making a new measurement).
3. Repeat for three different values of R.
4. Calculate $q = CV$ for the charge deposited in the capacitor and plot q_n vs n.

PART D : Electromagnetic damping

1. Let the coils be open. There will be no electromagnetic damping during the oscillations. Give a small (~ 20°) displacement to the metal arc and measure the amplitude θ_{An} after n swings (n = 1, 2, 3…).

   Plot $ln\theta_{An}$ vs. n and calculate the quality Q_0 (without emf damping). Repeat for another initial displacement.
2. Now connect the coils in series (B to C) and short the ends A & D. Measure the amplitudes θ_{An} (n = 1, 2, 3…) for the same two values of initial displacement as in step 1. Plot $ln\ \theta_{An}$ *vs.* n (it will be useful to plot all the four graphs on the same graph sheet) and obtain the value of quality Q with electromagnetic damping.

   Note : Since damping is small you may have to take measurement for a large (~10) number of swings.

III Exercises and Viva Questions

1. What is the advantage in having a large number of turns in the coil? What is the effect of connecting the two coils in series or in parallel?
2. Show that the angular frequency for small oscillations of the metal farm is given by $\sqrt{\frac{Mgl}{I}}$.
3. What is the effect of moving the weights closer to the point of suspension? Would the emf be the same for the same release angle ?
4. Find a way of estimating the angle/position at which the maximum emf occurs.
5. Estimate the pulse width τ for a given θ_0 (from your observation). Compare it with the time constant *RC*.
6. The charge deposited per swing (eq 9) appears to be constant (depending only on total change of flux and not on the velocity or θ_0). Does your observation of q depend on θ_0? If so why ?
7. What is the function of the diodes in this experiment ? What would happen if the diode were absent in the circuit ?
8. Give reasons why the graph of ln θ_{An} vs. n (Part D) could deviate from a straight line ?
9. Verify by substitution that $\theta(t) = \theta_0 e^{-\frac{\omega_0 t}{2Q}} cos(\omega t + \alpha)$ ($\omega = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$) is a solution of the differential equation (10).
10. Give some practical applications of Faraday's law.

References :

1. "Physics", M. Alonso and E.J. Finn, Addison Wesley, 1992.
2. "Introduction to Electrodynamics", D.J. Griffiths, PHI, 1998.

Experiment 5

Planck's Constant

Apparatus:

Photoelectric cell, *DC* source, *DC* milliammeter, Variac (*AC*) (0-260V), *AC* ammeter, Tungsten filament lamp (60 W), Monochromatic filters.

Purpose of experiment:

To measure the value of Planck's constant '*h*'.

Basic methodology:

Light from a tungsten filament lamp (assumed to be a black body source) is passed through a monochromatic filter and made to fall on a photoelectric cell. The slope of the graph $\ln\ I_{ph}\ vs.\ \frac{1}{T}$, leads to a determination of Planck's constant.

I Introduction:

I.1 The electromagnetic radiation emitted by a black body (a perfect absorber and emitter of electromagnetic radiation) is spread continuously over the entire electromagnetic spectrum. It was Planck, who first gave the law for black body radiation based on the idea that electromagnetic radiation is composed of quanta called photon of energy $\varepsilon = h\nu$, where ν is the frequency of radiation and *h* is Planck's constant.

I.2 Planck's law for radiation from a black body gives the spectral energy density of the radiation in the frequency range ν to $\nu + d\nu$. This is denoted as $U(\nu)\, d\nu$ and is given by

$$U(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)}\, d\nu \qquad (1)$$

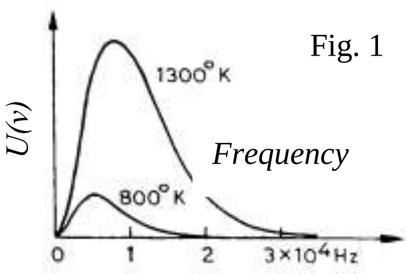


Fig. 1

In eq. (1), $c = 3 \times 10^8$ m/s is the speed of light, $k = 1.38 \times 10^{-23}$ J/K is the Boltzman constant. Fig. 1 shows a graph of $U(\nu)$ vs. ν for given temperatures *T*.

In the high frequency region, where $\frac{h\nu}{kT} \rangle\rangle 1$, eq. (1) can be approximated as

$$U(\nu) = \frac{8\pi h\nu^3}{c^3}\; e^{-\frac{h\nu}{kT}} \tag{2}$$

showing an exponential decrease in the energy density with frequency.

I.3 In this experiment, a tungsten filament lamp is taken to be a black body radiator. Using a monochromatic filter, radiation with frequency in the visible region is selected. For the range of temperatures of the tungsten filament, the energy density can be taken to be given by eq.(2). The energy density at the chosen frequency is indirectly measured by measuring the photocurrent I_{ph} generated upon exposing a photocell to the radiation. From the properties of the photoelectric effect, it is known that the photocurrent is proportional to the intensity of the radiation. Thus

$$I_{ph} \propto U(\nu) \approx \frac{8\pi h\nu^3}{c^3} e^{-\frac{h\nu}{kT}} \tag{3}$$

or

$$\ln I_{ph} = -\frac{h\nu}{kT} + ln\frac{8\pi h\nu^3}{c^3} = -\frac{h\nu}{kT} + \quad constant \tag{4}$$

Hence the graph of $ln\, I_{ph}$ vs. $1/T$ is a straight line of slope of magnitude $h\nu/k$.

I.4 The temperature of the tungsten filament can be varied by changing the current through it. The temperature of the filament can be estimated by measuring the resistance R of the filament. The variation of R with temperature for tungsten is given by the empirical formula (T is expressed in ^oC)

$$R = R_o\,(\,1 + \alpha T + \beta T^2\,) \tag{5}$$

Where,

$\alpha = 5.24 \times 10^{-3}\ (°C)^{-1}$

$\beta = 0.70 \times 10^{-6}\ (°C)^{-2}$.

A calibration graph can be obtained by drawing the graph of eq. (5). Then, knowing the resistance, $R = \frac{V}{I}$ of the filament the temperature T(°C) can be obtained from the calibration graph.

II. Set-up and Procedure

1. Complete the circuit with Tungsten lamp and photocell as shown in Fig. 2
2. Choose and set the color of the monochromatic filter (say red).
3. Using the variac, vary the *AC* voltage to the tungsten filament from 80V to 220 V in steps of 20 V.
4. Measure the *AC* current to the tungsten and the *DC* photocurrent I_{ph}.
5. Repeat the measurements for three filters in all (say red, blue and green).
6. Prepare the calibration graph of R (resistance of filament) by using eq. 5 to calculate R for value of T (^{o}C) = 400, 600, 800, 1000, …, 2000.

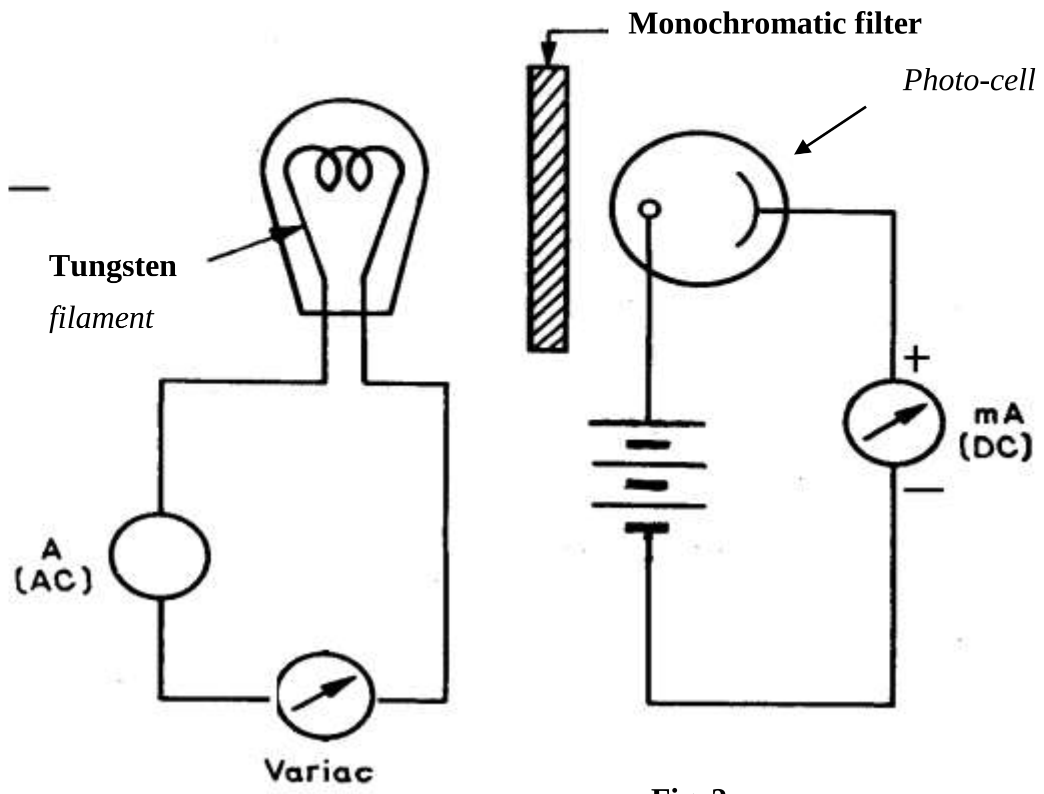


Fig. 2

Plot and fit the calculated value of R vs. T by a best-fit straight line.

7. Calculate the resistance $R = \frac{V}{I}$ from your measurement and use the calibration graph to read off temperature of the filament, against the value of the resistance.

Exercises and Viva Questions:

1. What is the meaning of the quantity $U(\nu)$ in Planck's black body radiation law ?
2. Give the approximate forms for the energy density for $\frac{h\nu}{kT} << 1$ and $\frac{h\nu}{kT} >> 1$. and corresponding laws.
3. What is the purpose of using a photocell in this experiment?
4. Use the energy density expression to argue how the photocurrent should change upon varying the frequency, keeping the variac voltage the same. How will I_{ph} change if frequency is kept constant but the variac voltage is varied? Verify your expectations from your observations.
5. Argue how I_{ph} would change as the variac voltage is changed if the tungsten lamp were allowed to illuminate the photocell without using a filter in between. What frequency would contribute most to the photocurrent?
6. Study the photoelectric effect and list the characteristics of the photoelectric effect that can only be explained by the quantum nature of light.

7. Is our assumption that $I_{ph} \propto U(\nu)$ always right ? Is it true that radiation of any frequency will give rise to a photocurrent?
8. Look up the value of work function of tungsten and calculate the cut off frequency ν_0 for tungsten.
9. We have taken the tungsten filament to be a black body radiator. What qualitative changes should we expect if it were to be taken to be an imperfect black body radiator?
10. What is the significance of Planck's constant in Physics?

References :

1. “Physics”, M. Alonso and J. Finn, Addison Wesley 1992.
2. “Modern Physics”, A. Beiser, McGraw Hill Inc., 1995

<u>Experiment 6</u>
Newton's Rings

Apparatus:

Traveling microscope, sodium vapor lamp, plano-convex lens, plane glass plate, magnifying lens.

Purpose of experiment:

To observe Newton's rings formed by the interference produced by a thin air film and to determine the radius of curvature of a plano-convex lens.

Basic methodology :
A thin wedge shaped air film is created by placing a plano-convex lens on a flat glass plate. A monochromatic beam of light is made to fall at almost normal incidence on the arrangement. Ring like interference fringes are observed in the reflected light. The diameter of the rings are measured.

I. Introduction

I.1 The phenomenon of Newton's rings is an illustration of the interference of light waves reflected from the opposite surfaces of a thin film of variable thickness. The two interfering beams, derived from a monochromatic source satisfy the coherence condition for interference. Ring shaped fringes are produced by the air film existing between a convex surface of a long focus plano-convex lens and a plane of glass plate.

I.2 ***Basic theory:***

When a plano-convex lens (*L*) of long focal length is placed on a plane glass plate (*G*), a thin film of air is enclosed between the curved surface of the lens and upper surface of the glass plate (See Fig. 1). The thickness of the air film is very small at the point of contact and gradually increases from the centre out-wards. The fringes produced are concentric circles. With monochromatic light, bright and dark circular fringes are produced in the air film. When viewed with white light, the fringes are colored.

A horizontal beam of light falls on the glass plate *B* at an angle of 45°. The glass plate *B* reflects a part of the incident light towards the air film enclosed by the lens *L* and plate *G*. The reflected beam (See Fig. 1) from the air film is viewed with a microscope. Dark and bright circular fringes are observed. This is due to the interference between the light reflected at the curved surface of the lens and the upper surface of the plate *G*.

For normal incidence the optical path difference between the two interfering waves is nearly $2\mu t$, where μ is the refractive index of the film and t the thickness of the film. Here an extra phase difference of π occurs for the ray, which has got reflected from upper surface of the plate G because the incident beam in this reflection goes from a rarer to a denser medium. Thus, the conditions for constructive

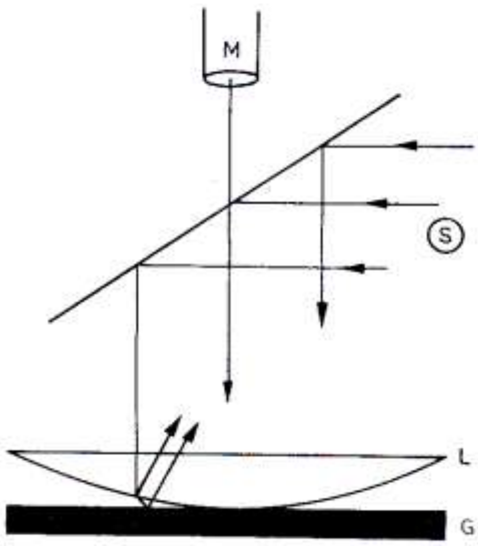


Fig. 1

and destructive interference are (using μ = for air),

$$2t = m\lambda \quad for \ minima; \ m = 0,1,2... \qquad (1)$$

and

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad for \ maxima; \ m = 0,1,2... \qquad (2)$$

The air film enclosed between the spherical surface of radius R and a plane surface glass plate, gives circular rings such that

(see Fig. 2) $r_m^2 = (2R-t)\,t.$

where r_m is the radius of the m^{th} order dark ring. (Note: The m^{th} order dark ring is the m^{th} dark ring excluding the central dark spot).

Now R is of the order of 100 cm and t is at-most 1 cm . Therefore $R >> t$. Hence

$$(R - t)^2 + r_m^2 = R^2 \ \Rightarrow r_m^2 = (2R - t)\,t$$

$$2t \approx \frac{r_m^2}{R}$$,(neglecting the t^2 term),.

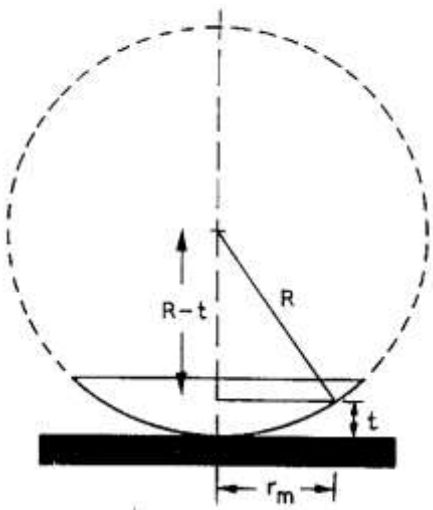


Fig. 2

Putting the value of $2t$ in eq. (1) gives,

$$m\lambda \approx \frac{r_m{}^2}{R} \quad \Rightarrow r_m{}^2 \approx m\lambda R, \qquad m = 0,1,2... \tag{3}$$

and eq. (2) gives (for the radius r_m of m^{th} order bright ring)

$$\frac{r_m{}^2}{R} = (m + \frac{1}{2})\lambda \Rightarrow r_m{}^2 = \left(m + \frac{1}{2}\right)\lambda R. \tag{4}$$

Hence, for dark rings

$$r_m = \sqrt{m\lambda R} \tag{5}$$

while for bright rings

$$r_m = \sqrt{\left(m + \frac{1}{2}\right)\lambda R} \;; \quad m = 0,1,2... \tag{6}$$

With the help of a traveling microscope , we can measure the diameter of the m^{th} order dark ring $= D_m$, Then $r_m = \frac{D_m}{2}$ and hence,

$$D_m{}^2 = 4m\lambda R. \tag{7}$$

So if we know the wavelength λ, we can calculate R (radius of curvature of the lens).

II Setup and procedure:

1. Clean the plate *G* and lens *L* thoroughly and put the lens over the plate with the curved face resting on the glass plate (see Fig 1).
2. Switch on the monochromatic light source. This sends a parallel beam of light. This beam of light gets reflected by plate *B* falls on lens *L*.
3. Look down vertically from above the lens and see whether the centre is well illuminated. On looking through the microscope, a spot with rings around it can be seen on properly focusing the microscope.
4. Once good rings are in focus, rotate the eyepiece such that out of the two perpendicular cross wires, one has its length parallel to the direction of travel of the microscope. Let this cross wire also pass through the center of the ring system.
5. Now move the microscope to focus on a ring (say, the 20th order dark ring). On one side of the centre. Set the crosswire tangential to one ring as shown in Fig 3. Note down the microscope reading.

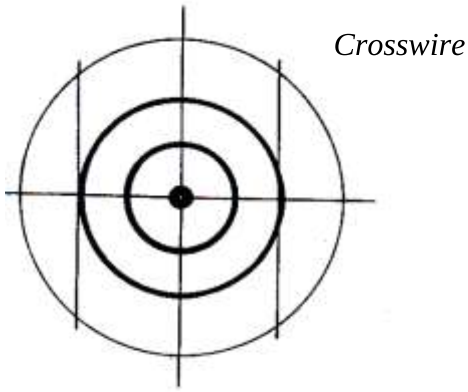


Fig. 3

(Make sure that you correctly read the least count of the vernier in mm units).

6. Move the microscope to make the crosswire tangential to the next ring nearer to the centre and note the reading. Continue with this process till you pass through the center. Take readings for an equal number of rings on the both sides of the centre.

Precautions:

Notice that as you go away from the central dark spot, the fringe width decreases. In order to minimize errors in measurement of the diameter of the rings, the following precautions should be taken:

- *i* *The microscope should be parallel to the edge of the glass plate.*
- *ii* *If you place the cross wire tangential to the outer side of a particular ring on one side of the central spot then the cross wire should be placed tangential to the inner side of the same ring on the other side of the central spot. (See Fig. 3)*
- *iii* *The traveling microscope should move only in one direction.*

III Exercises and Viva Questions:

1. What is the medium that causes the interference in this experiment? Why are the interference effect due to the glass plate and the lens ignored ?
2. Explain why the interference rings are a circular in shape.
3. Why do the rings get closer as the order of the rings increases ?
4. Show that the difference in radius between adjacent bright ring is given by

$$\Delta r = r_{m+n} - r_m \approx \frac{1}{2}\sqrt{\frac{\lambda R}{m}} \quad \text{for } m >> 1.$$

5. Show that the area between adjacent ring is independent of m and is given by

   $A = \pi\lambda R$, for $m >> 1$.

6. Why is the central spot dark ? What would be the reason for not obtaining a dark central spot in the experiment?
7. What would be the shape of the rings if a wedge shaped prison went kept inverted on the glass plate.
8. What will be the effect of using a plano convex lens in the experiment?

   Derive the expression for the radius of bright and dark fringes.

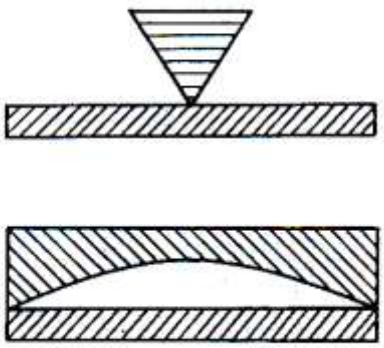


Fig. 4

9. What would be effect of using white light instead of a monochromatic light ?
10. Why is it necessary to use a lens of large value of R in this experiment ?

Reference:

1. "Physics", M. Alonso and E.J. Finn, Addison Wesley 1992.
2. "Fundamental of Physics", D. Halliday, R. Resnick and J. Walker, John Wiley & Sons, New York, 2001.

Experiment 7

Diffraction at a single and double slit

Apparatus:

Optical bench, He – Ne Laser, screen with slits, photocell, micro-ammeter.

Purpose of the experiment:

To measure the intensity distribution due to diffraction due to single and double slits and to measure the slit width (d) and slit separation (a).

Basic methodology:

Light from a He – Ne Laser source is diffracted by single and double slits. The resulting intensity variation is measured by a photocell whose output is read off as a current measurement.

I. Introduction:

I.1 Single slit diffraction

We will study the Fraunhofer diffraction pattern produced by a slit of width 'a'. A plane wave is assumed to fall normally on the slit and we wish to calculate the intensity distribution produced on the screen. We assume that the slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets which interfere with the wavelets emanating from other secondary points. Let the point sources be at A_1, A_2, A_3 … and let the distance between the consecutive points be Δ. (See Fig. 1). Thus, if the number of point

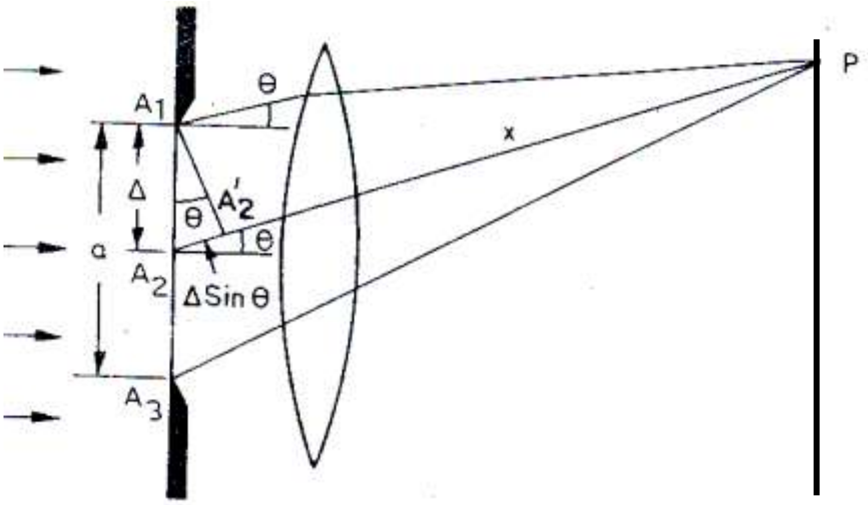


Fig.1

sources be n, then.

$$a = (n-1)\Delta. \qquad (1)$$

We now calculate the resultant field produced by these n sources at point P on the screen. Since the slit actually consists of a continuous distribution of sources, we will in the final expression, let n go to infinity and Δ go to zero such that $n\Delta$ tends to a.

Now at point P the amplitudes of the disturbances reaching from A_1, A_2,... will be very nearly the same because the point P is at a distance which is very large in comparison to a. However, because of even slightly different path lengths to the point P, the field produced by A_1 will differ in phase from the field produced by A_2.

For an incident plane waves, the points A_1, A_2... are in phase and, therefore, the additional path traversed by the disturbance emanating from the point $A_2 A_2'$. This follows from the fact that the optical paths A_1B_1P and $A_2'B_2P$ are the same. If the diffracted rays make an angle θ with the normal to the slit the path difference would be

$$A_2 A_2' = \Delta \sin\theta. \tag{2}$$

The corresponding phase difference, ϕ, would be given by

$$\phi = \frac{2\pi}{\lambda} \Delta \sin\theta \tag{3}$$

Thus, if the field at the point P due to the disturbance emanating from the point A_1 is $a\cos(\omega t)$ then the field due to the disturbance emending from A_2 would be emanating $a\cos(\omega t - \phi)$. Now the difference in phases of the disturbance reaching from A_2 and A_3 will also be φ and thus the resultant field at the point P would be given by

$$E = E_0\left[\cos\omega t + \cos(\omega t - \phi) + ... + \cos(\omega t - (n-1)\phi)\right] \tag{4}$$

Because

$\cos \omega t + \cos(\omega t - \phi) + \ldots + \cos[\omega t - (n-1)\phi]$

$$= \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \tag{5}$$

Thus,

$$E = E_\theta \cos\left[\omega t - \frac{1}{2}(n-1)\phi\right] \tag{6}$$

Where the amplitude E_θ of the resultant field would be given by

$$E_\theta = \frac{E_0 \sin\left(\frac{n\phi}{2}\right)}{\sin\frac{\phi}{2}} \tag{7}$$

In the limit of $n \to \infty$ and $\Delta \to 0$ in such a way that $n\Delta \to a$, we have

$$\frac{n\phi}{2} = \frac{n}{2} \cdot \frac{2\pi}{\lambda} \Delta \sin\theta \to \frac{\pi}{\lambda} a \sin\theta$$

Further $\phi = \frac{2\pi}{\lambda}\Delta sin\theta = \frac{2\pi}{\lambda}\frac{a}{n}sin\theta$ would tend to zero and we may therefore, write

$$E_\theta = \frac{E_0 \; sin\frac{n\phi}{2}}{\frac{\phi}{2}} = n\,E_0 \frac{sin\left(\frac{\pi a\,sin\theta}{\lambda}\right)}{\frac{\pi}{\lambda}a\,sin\theta} = A\,\frac{sin\,\beta}{\beta} \tag{8}$$

where,

$$A = n\,E_0 \quad \text{and} \quad \beta = \frac{\pi\,a\,sin\theta}{\lambda}. \tag{9}$$

Thus,

$$E = A\frac{sin\,\beta}{\beta}\,cos(\omega t - \beta) \tag{10}$$

The corresponding intensity distribution is given by

$$I = I_0\,\frac{sin^2\,\beta}{\beta^2} \tag{11}$$

Where I_0 represent the intensity at $\theta = 0$.

I.2 Positions of the maxima and minima:

The variation of the intensity with β is shown in Fig 2a. From eq. (11) it is obvious that the intensity is zero when

$$\beta = m\pi, \qquad m \neq 0 \tag{12}$$

or

$$a\;sin\;\theta = m\lambda\,; \qquad m = \pm 1\;, \pm 2\;, \pm 3 \quad (minima)$$

In order to determine the position of the maxima, we differentiate eq. (11) wrt. β and set it equal to zero.

This gives

$$tan\,\beta = \beta \quad (maxima) \tag{13}$$

The root β = 0 corresponds to the central maximum. The other roots can be found by determining the points of intersections of the curves $y = \beta$ and $y = tan\,\beta$ (Fig 2b,c).

The intersections occur at β = 1.43 π, β = 2.46 π etc. and are known as the first, second maximum etc. Since $\left[\frac{sin(1.43\pi)}{1.43\pi}\right]^2$ is about 0.0496, the intensity of the first maximum is about 4.96% of the central maxima. Similarly, the intensities of the second and third maxima are about 1.88% and 0.83% of the central maximum respectively.

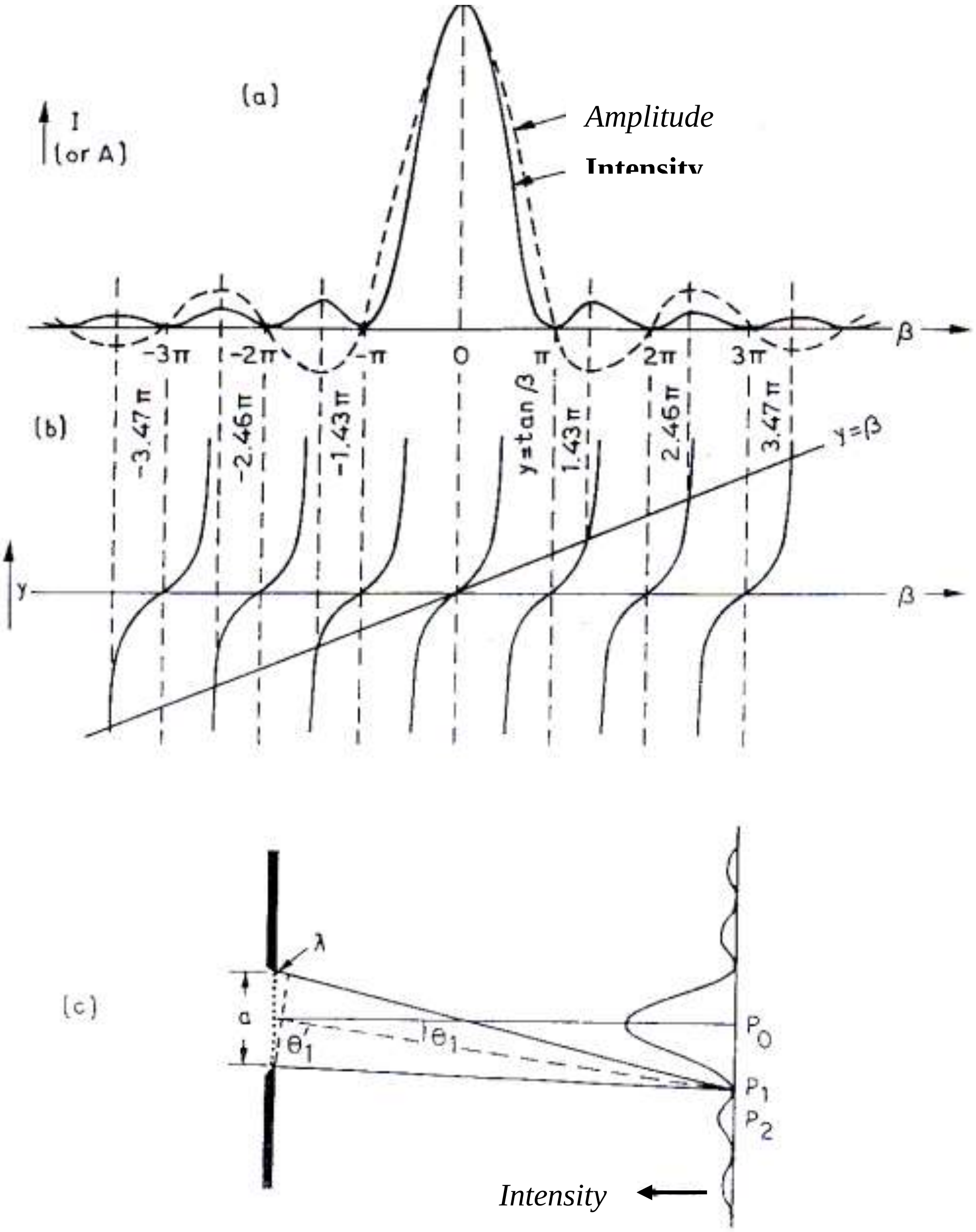


Fig. 2

I. 3 Double slit diffraction pattern:

In this section we will study the Fraunhofer diffraction pattern produced by two parallel slits (each of width *a*) separated by a distance *d*. We would find that the resultant intensity distribution is a product of single slit diffraction pattern and the interference pattern produced by two point sources separated by a distance *d*.

In order to calculate the diffraction pattern we use a method similar to that used for the case of a single slit and assume that the slits consist of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets. Let the point sources be at A_1 , A_2 , A_3 …(in the first slit) and at B_1, B_2, B_3…(in the second slit) (see Fig 3). As before, we assume that the distance between two consecutive points in either of the slit is Δ. Then the path difference between the disturbances reaching the point *P* from two consecutive point in a slit will

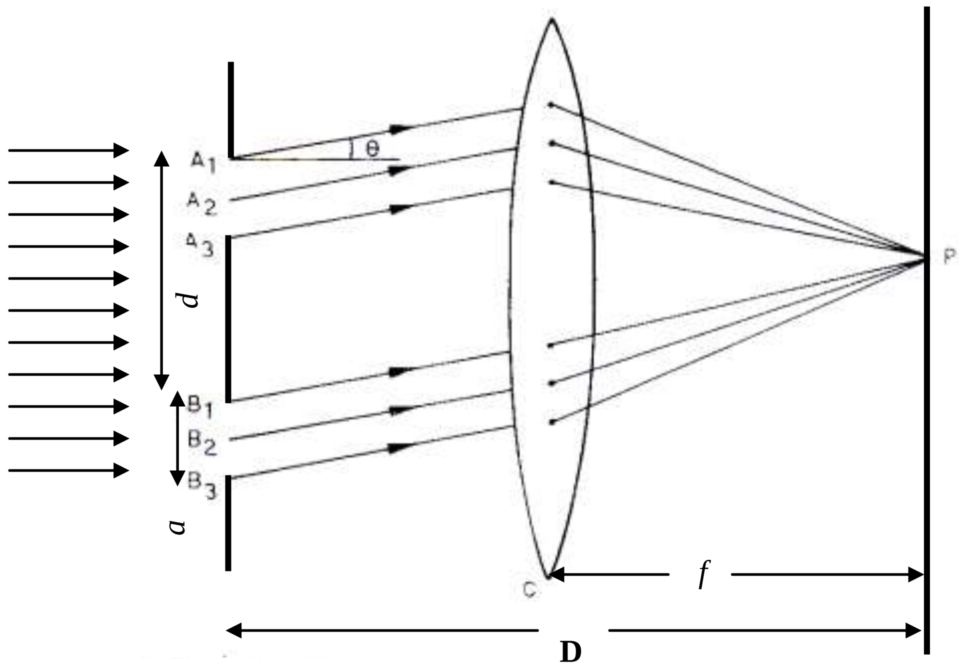


Fig. 3

be $\Delta \sin \theta$. The field produced by the first slit at the point P will, therefore, be given by (see eq. 10).

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \tag{14}$$

Similarly, the second slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \phi_1) \tag{15}$$

at the point *P*, where $\phi_1 = \frac{2\pi}{\lambda} d \sin\theta$ represents the phase difference between the disturbances from two corresponding points on the slits; by corresponding points we imply pair of points like (A_1, B_1) , (A_2 , B_2) …, which are separated by a distance d. Hence the resultant field will be

$$E = E_1 + E_2 = A\frac{\sin\beta}{\beta}\left[\cos(\omega t-\beta) + \cos(\omega t-\beta-\phi_1)\right]$$

which represents the interference of two waves each of amplitude $A\frac{\sin\beta}{\beta}$ and differing in phase by ϕ_1. Above equation can be rewritten as

$$E = A\frac{\sin\beta}{\beta}\ \cos\gamma\ \cos\ (\omega t-\beta-\frac{\phi_1}{2})$$

Where $\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda}d\ \sin\theta$.

The intensity distribution will be of the form

$$I = 4 I_0 \frac{\sin^2\beta}{\beta^2}\cos^2 r \qquad (16)$$

where $I_0 \frac{\sin^2\beta}{\beta^2}$ represents the intensity distribution produced by one of the slits. As can be seen, the intensity distribution is a product of two terms, the first term $\left(\frac{\sin^2\beta}{\beta^2}\right)$ represents the diffraction pattern produced by a single slit of width a and the second term ($\cos^2\gamma$) represents the interference pattern produced by two point sources separated by a distance d (see Fig 4).

I.4 Positions of Maxima & Minima:

Equation (16) tells us that the intensity is zero wherever $\beta = \pi, 2\pi, 3\pi, \ldots$

or when $\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

The corresponding angles of diffraction will be given by

$$a\sin\theta = m\lambda\ ;\ (m = 1, 2, 3, \ldots)$$

and

$$d\,sin\theta = \left(n + \frac{1}{2}\right)\lambda; \qquad (n = 0,1,2,3\ldots) \qquad (17)$$

Interference maxima occur when

$$\gamma = 0,\ \pi,\ 2\pi, \ldots.$$

or when

$$d\ sin\ \theta = 0,\ \lambda,\ 2\ \lambda,\ 3\lambda\ \ldots$$

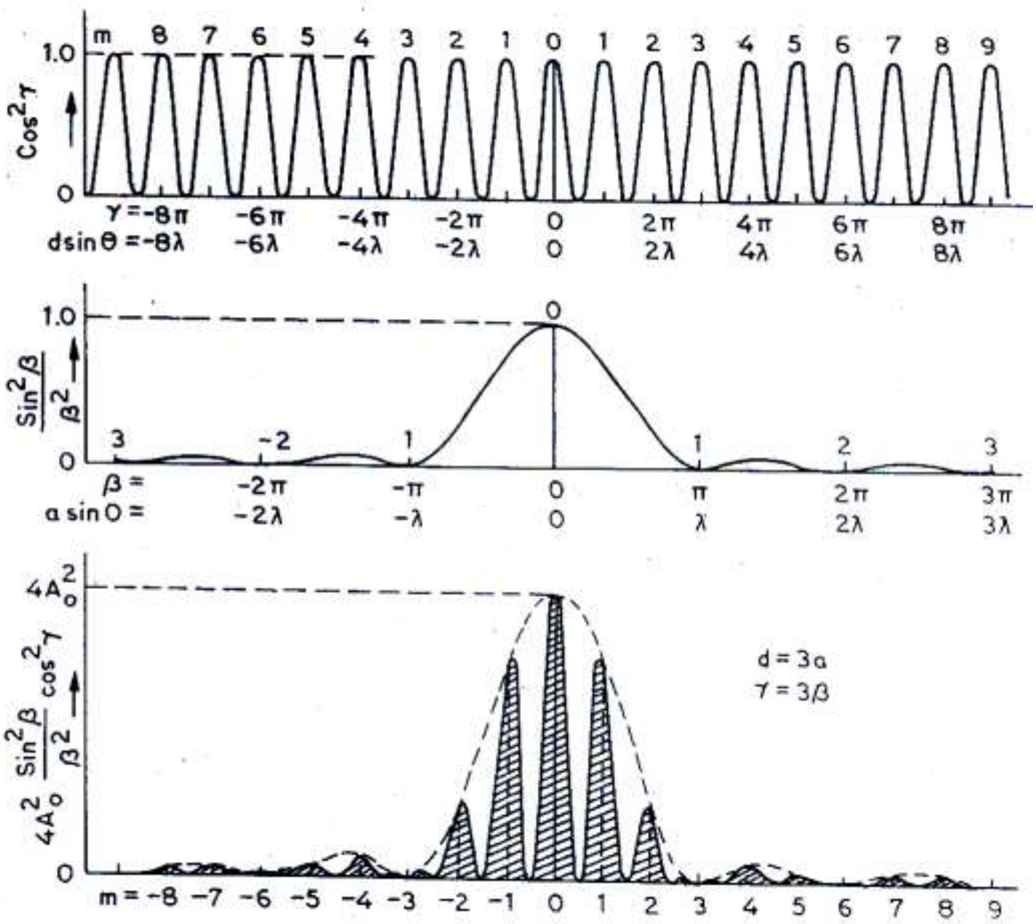


Fig. 4

II Set-up and procedure:

1. Switch on the laser source about 15 minutes before the experiment is due to start. This ensures the intensity of light from the laser source is constant.
2. Allow the laser beam to fall on a single slit formed in the screen provided.
3. The intensity distribution in the diffraction pattern is measured with the help of a photocell. The photocell is secured to a mount and is kept as far behind the slit as possible. A screen with a slit (0.3mm wide) is fitted in front of the photocell. The photocurrent is measured with a multimeter (μA) range and is approximately proportional to intensity of the incident light.

4. Repeat the same procedure for double slit and record the diffraction pattern on both the sides of central maximum. The interval between two consecutive. minima position of the photocell should be small enough, so that adjacent maxima/minima of the intensity distribution are not missed.

Precautions:

1. *The laser beam should not penetrate into eyes as this may damage the eyes permanently.*
2. *The photocell should be as away from the slit as possible.*
3. *The laser should be operated at a constant voltage 220V obtainable from a stabilizer. This avoids the flickering of the laser beam.*

III Exercises and Viva & Questions

1. What are the characteristics of light produced by a laser? Can this experiment conducted by using any other source?
2. Verify eq. 4
3. For a travelling wave, derive the relation between path difference and phase differences.
4. What is the effect on the intensity distribution if the slit of width 'a' is changed? If the slit separation d is changed?
5. What would be the result if the experiment were to be carried out with white light?
6. What is the intensity distribution for a double slit ignoring diffraction effects.
7. Count the number of interference fringes observed within the envelope of central diffraction maximum. Give an explanation based on the experiment for the number of fringes seen.
8. What is the effect on the intensity pattern if the distance D between slit and photocell is changed?
9. How much should D change for a bright fringe at the photocell to be replaced by dark fringe?
10. What will be the intensity pattern for a 3-slit interference?

References :

1. "Fundamental of Optics", F.A. Jenkins and H.E. White, McGraw-Hill International (4th edition), 1976.
2. "Optics", A. Ghatak, Tata McGraw-Hill (2nd edition), 1992.
3. "Fundamentals of Physics", D. Halliday, R. Resnick and J.A. Walker, John Wiley & Sons, 2001.

Appendix : Lasers

Introduction:

The light emitted form a conventional light source (like a sodium lamp) is said to be incoherent because the radiation emitted from different atoms do not, in general, bear any definite phase relationship with each other. On the other hand, the light emitted from a laser has a very high degree of coherence and is almost perfectly collimated.

Laser is an acronym for Light Amplification by Stimulated Emission of Radiation. The basic principle involved in lasing action is the phenomenon of stimulated emission, which was predicted by Einstein in 1917. Einstein argued that when an atom is in the excited state, it can make a transition to a lower energy state through the emission of electromagnetic radiation; however, in contrast to the absorption process, the emission can be occur in two different ways:

i. The first is referred to as *spontaneous emission* in which an atom in the excited state emits radiation even in the absence of any incident radiation. It is thus not stimulated by any incident signal but occurs spontaneously.

ii. The second is referred to as *stimulated emission* in which an incident signal of appropriate frequency triggers an atom in an excited state to emit radiation.

Using the phenomenon of stimulated emission, C.H. Townes and A.H. Schawlow, in 1958, worked out the principle of the laser.

Stimulated Emission:

Consider a gas enclosed in a vessel containing free atoms having a number of energy levels, at least one of which is *metastable*. By shining white light into this gas many atoms can be exited, through resonance, from the ground state to excited states. As the electrons drop back, many of them will become trapped in the metastable states. If the pumping light is intense enough we may obtain a population inversion, i.e. more electrons in the metastable state than in the ground state.

When an electron in one of these metastable states spontaneously jumps to the ground state, it emits a photon. As the photon passes by another nearby atom in the same metastable state, it stimulates that atom to radiate a photon of the exact same frequency, direction, and polarization as the primary photon and exactly the same phase. Both of these photons upon passing close to other atoms in their metastable states, stimulate them to emit in the same direction with the same phase. However, transitions from the ground state to the exited state can also be stimulated thereby absorbing the primary photons. An excess of stimulated emission gives population inversion. Thus if the conditions in the gas are right, a chain reaction can be developed, resulting in high intensity coherent radiation.

Laser Design:

In order to produce a laser, one must collimate the stimulated emission, and this is done by properly designing a cavity in which the waves can be used over and over again. For this, a cavity is attached with two end mirrors with high reflecting power and into this cavity is introduced an appropriate solid, liquid, or gas having metastable states in the atoms or molecule. The electrons in these atoms are exited and produce a population inversion of atoms in a metastable state, which spontaneously radiate. Photons moving at an appreciable angle to the walls of the cavity will escape and be lost. Those photons emitted parallel to the axis will reflect back and forth from end to end. Their chance of stimulating emission will now depend on the high reflectance at the end mirrors and a high population density of metastable atoms within the cavity. If both these conditions are satisfied the build-up of photons surging back and forth through the cavity can be self sustaining and the system will oscillate or lase, spontaneously.

Helium – Neon Gas Laser:

The He-Ne laser was first fabricated by Al Jaran, Bennett and Harriott in 1961 at Bell Telephone laboratories in USA. This consists of a mixture of helium and neon gases in a ratio of about 10:1, placed inside a long narrow discharge tube. (See Fig.5). The pressure inside the tube is 1 mm of Hg. The gas system is enclosed between a pair of plane mirrors. One of the mirrors is of very high reflectivity while the other is partially transparent so that energy may be coupled out of the system.

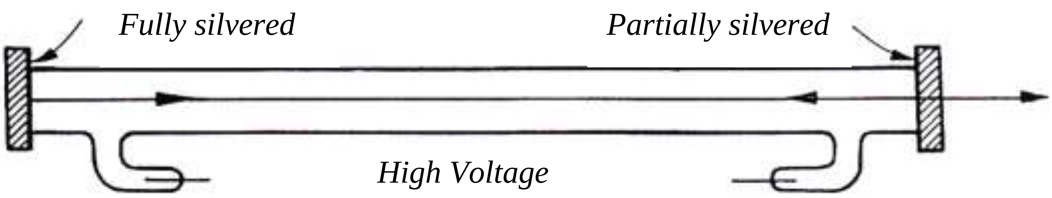


Fig. 5

All the lower energy levels of He and Ne are shown in an energy level diagram in Fig. 6. The normal state of helium is $^1S_0$ level arising from two valence electrons in *1s* orbits. The excitation of either one of these electrons to the *2s* orbit finds the atom in a $^1S_0$ or a $^3S_1$ state, both quite metastable, since transitions to the normal state are forbidden by selection rules.

Neon, with Z = 10, has 10 electrons in the normal state and is represented by the configuration $1s^2\ 2s^2\ 2p^6$. When one of the six *2p* electrons is excited to the *3s, 3p, 4s, 4p, 4d, 4f, 5s*, etc., orbit, triplet and singlet energy levels arise. A subshell like $2p^5$, lacking only one electron from a closed subshell, behaves as though it were a subshell containing one $2p^5$ electron. The number and designations of the levels produced are therefore the same as for two electrons, all triplets and singlets.

As free electrons collide with helium atoms during the electric discharge, one of the two bound electrons may be excited to *2s* orbits, i.e., to the $^3S_1$ *or* $^1S_0$ states. Since downward transitions are forbidden by radiation selection rules, these are metastable states and the number of excited atoms increases. We therefore have optical pumping, out of the ground state $^1S_0$ and into the metastable states $^3S_1$ *and* $^1S_0$.

When a metastable helium atom collides with a non atom in its ground state, there is a high probability that the excitation energy will be transferred to the neon, raising it to in one of the $^1P_1$ *or* $^3P_0$, $^3P_1$, or $^3P_2$ levels of $2p^55s$. The small excess energy is converted into kinetic energy of the colliding atoms.

In this process each helium atom returns to the ground state as each colliding neon atom is excited to the upper level of corresponding energy. The probability of a neon atom being raised to the $2p^53s$ *or* $2p^53p$ levels by collision is extremely small because of the large energy mismatch. The collision transfer therefore selectively increases the population of the upper levels of neon.

Since selection rules permit transitions from these levels downward to the 10 levels of $2p^53p$ and these in turn to the 4 levels of $2p^53s$, stimulated emission can speed up the process of lasing. Lasing requires only that the *4s* and *5s* levels of neon be more densely populated than the *3p* levels. Since the *3p* levels of neon are more sparsely populated, lasing can be initiated without pumping a majority of the atoms out of the ground state. Light waves emitted within the laser at wavelengths such as 6328, 11,177 and 11,523 Å will occasionally be omitted parallel to the tube axis. Bouncing back and forth between the end mirrors, these waves will simulate emission of the same frequency from other excited neon atoms, and the initial wave with the stimulated wave will travel parallel to the axis. Most of the amplified radiation emerging from the ends of the He-Ne gas laser are in the near-infrared region of the

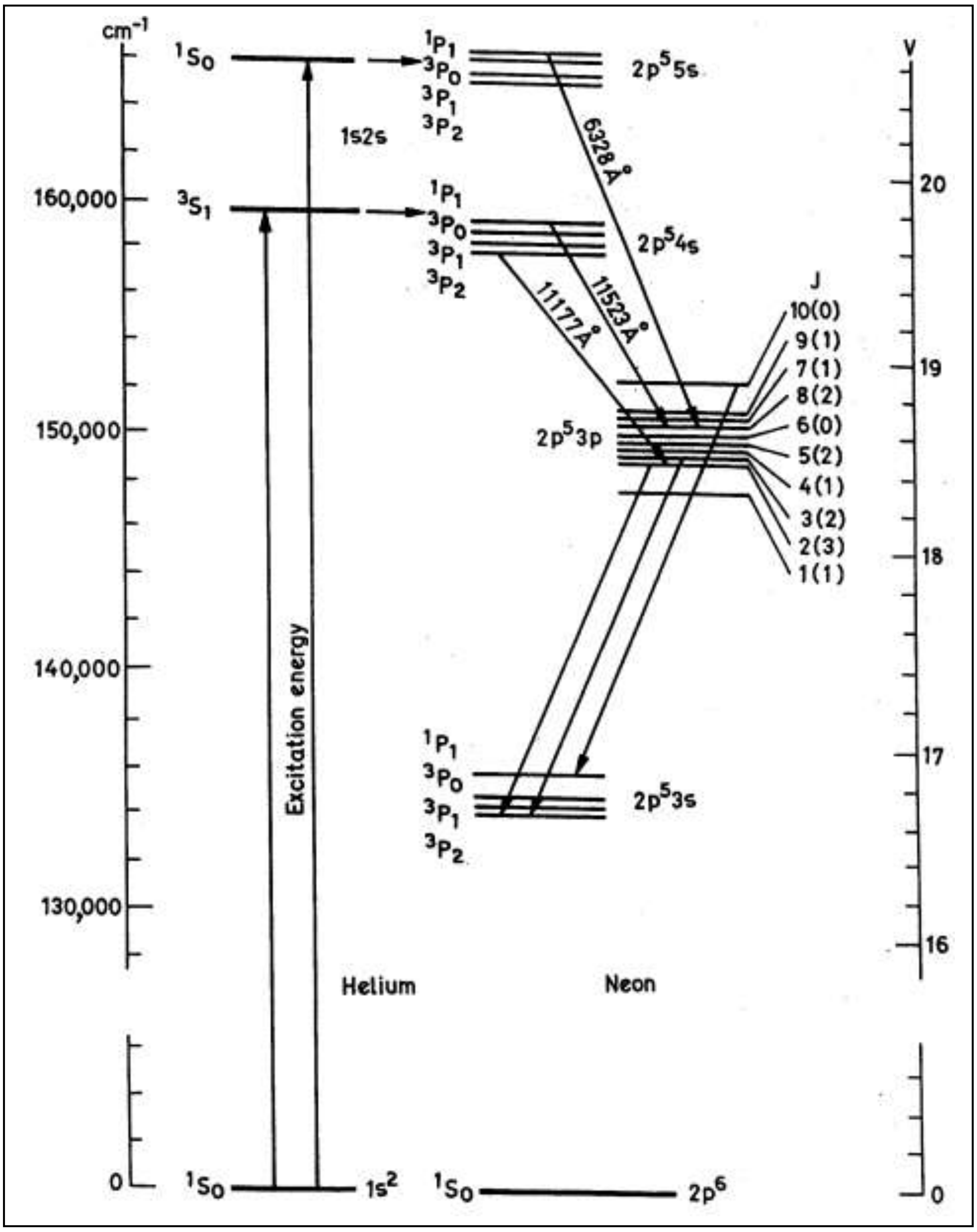


Fig. 6

spectrum, between 10,000 Å and 35,000 Å, the most intense amplified wavelength in the visible spectrum being the red line at 6328 Å.

Experiment 8

Diffraction Grating

Apparatus:

Spectrometer, grating, sodium lamp, mercury lamp, power supply for spectral lamps, magnifying glass.

Purpose of experiment:

i. To calibrate the grating spectrometer using the known source (Hg source) of light and to calculate the grating constant.

ii. Using the same grating, to calculate the wavelength of sodium – *D* lines.

Basic methodology:

Light from a mercury source is made to fall normally on a grating mounted on a spectrometer. The diffraction angle of the diffracted light is measured for each spectral line of the Hg-source. Likewise for sodium source**,** the diffraction angle and angular separation $\Delta\theta$ of the sodium doublets is measured.

I. Introduction:

I.1 *Diffraction Grating:*

A diffraction grating is a very powerful and precise instrument for the study of spectra and is widely used in a large number of fields from Astronomy to Engineering, wherever there is a need for detection of the presence of atomic elements.

A diffraction grating can be simply thought of as a set of identical and equally spaced slits separated by opaque strips. In reality gratings are made by ruling fine grooves by a diamond point either on a plane glass surface to produce a transmission grating or on a metal mirror to produce a reflection grating. In a transmission grating the grooves scatter light and so are opaque while the unruled surfaces transmit and act like slits. Typically a high quality grating (used for studying spectra in the visible range) has about 15000 grooves per inch, which gives a slit spacing of the order of a micron.

The chief requirement of a good grating is that the lines be equally spaced over the width of the ruled surface, which can vary from 1-25 cm. After each groove has been ruled, the machine lifts the diamond point and moves the grating forward by a small rotation of a screw. For rulings of equal spacing the screw must have a constant pitch. Replication gratings are also used, in which a cast of the ruled surface

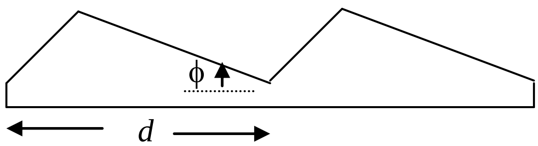


Fig. 1

is taken with some transparent material. Replication gratings give satisfactory performance where very high resolving power is not required. A typical groove profile is the triangular blazed profile shown in Fig. 1.The angle ϕ is called the blaze angle.

I.2 *Basic Theory:*

When a wave front is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. The secondary waves from the positions of the slit interfere with one another, similar to the interference of waves in Young's experiment. If the spacing between the lines is of the order of the wave length of light then an appreciable deviation of the light is produced.

Consider the diffraction pattern produced by *N* parallel slits, each of width b; the distance between two consecutive slits is assumed to be *d*. (See Fig.2).

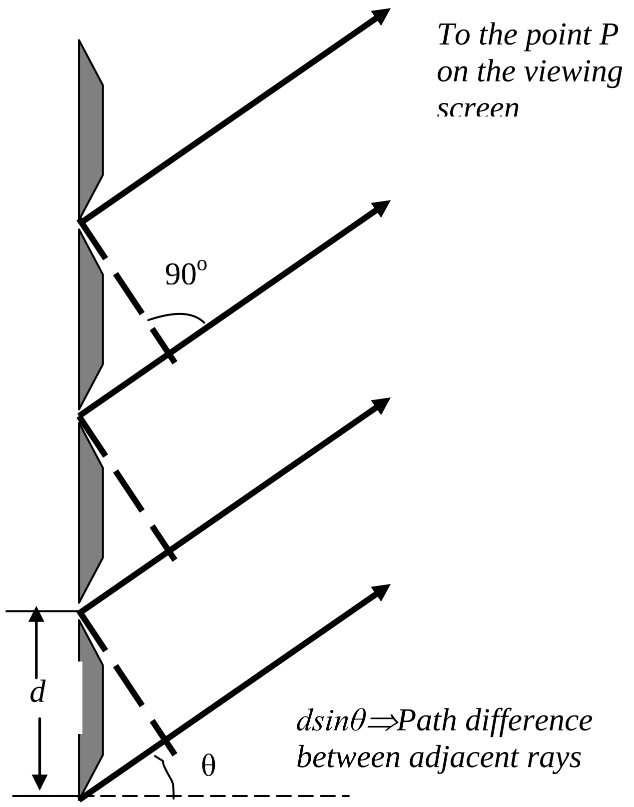


Fig. 2

The field at any arbitrary point *P* will essentially be a sum of *N* terms (recall the derivation for the double slit),

$$E = A\frac{\sin\beta}{\beta}\cos(\omega t-\beta) + A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-\phi_1)+\ldots + A\frac{\sin\beta}{\beta}\cos(\omega t-\beta-(N-1)\phi_1) \quad (1)$$

$$E = A\frac{\sin\beta}{\beta}\ [\cos(\omega t-\beta) + \cos(\omega t - \beta - \phi_1)+ \dots + \cos(\omega t - \beta - (N-1)\ \phi_1)]$$

$$= A\frac{\sin\beta}{\beta}\ \frac{\sin N\gamma}{\sin\gamma}\ \cos[\omega t - \beta - ½\,(N-1)\ \phi_1] \qquad (2)$$

where $\beta = \dfrac{\pi b \sin\theta}{\lambda}$ *and* $\gamma = \dfrac{\phi_1}{2} = \dfrac{\pi}{\lambda}\ d \sin\ \theta$

and ϕ_1 is the phase difference between the light rays emanating from successive slits. The corresponding intensity distribution will be

$$I = I_0\ \frac{\sin^2\beta}{\beta^2}\ \frac{\sin^2 N\gamma}{\sin^2\gamma} \qquad (3)$$

As can be seen, the intensity distribution is a product of two terms, the first term $\left(\dfrac{\sin^2\beta}{\beta^2}\right)$ represents the diffraction pattern produced by a single slit and the second term $\left(\dfrac{\sin^2 N\gamma}{\sin^2\gamma}\right)$ represents the interference pattern produced by N equally spaced slits. For $N = 1$ eq. (3) reduces to the single slit diffraction pattern and for $N = 2$, to the double slit diffraction pattern.

I.3 *Principal Maxima:*

When the value of N is very large, one obtains intense maxima at $\gamma = m\pi$ i.e., when

$$d \sin\theta = m\lambda, \quad m = 0, 1, 2,\dots \text{ (Maxima)} \qquad (4)$$

Thus, it can be easily seen by noting that

$$\lim_{\gamma \to m\pi} \frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma \to m\pi} \frac{N\cos N\gamma}{\cos\gamma} = \pm N\,.$$

Thus, the resultant amplitude will be

$$E(\theta) = \text{N A}\frac{\sin\beta}{\beta}$$

and the corresponding intensity distributions are given by

$$I = N^2 I_0\ \frac{\sin^2\beta}{\beta}, \text{ where } \beta = \frac{\pi b \sin\theta}{\lambda} = \frac{\pi b m}{d}$$

Such maxima are known as principal maxima. Physically, at these maxima the fields produced by each of the slits are in phase and, therefore, they add up and the resultant field is N times the field produced by each of the slits.

I.4 *Minima and secondary maxima:*

To find the minima of the function $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$ we note that the numerator becomes zero at $N\gamma = 0, \pi, 2\pi$ or in general, $p\pi$ where p is an integer. In the special case when $p=0, N, 2N...$, γ will be $0, \pi, 2\pi;...$ For these values the denominator will also vanish, and we have the principal maxima described above. The other values of p give zero intensity since for these the denominator does not vanish at the same time. Hence the condition for minima is $\gamma = p\pi/N$, excluding those values of p for which $p = mN$, m being the order. These values of γ corresponds to

$$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, ..., \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N} \tag{5}$$

Omitting the values 0, $\frac{N\lambda}{N}, \frac{2N\lambda}{N}...$, for which $d\sin\theta = m\lambda$ and which corresponds to principal maxima. Thus, between two principal maxima we have (N-1) minima. Between two such consecutive minima the intensity has to have a maxima; these maxima are known as secondary maxima. These are of much smaller intensity than principal maxima. The principal maxima are called spectrum lines.

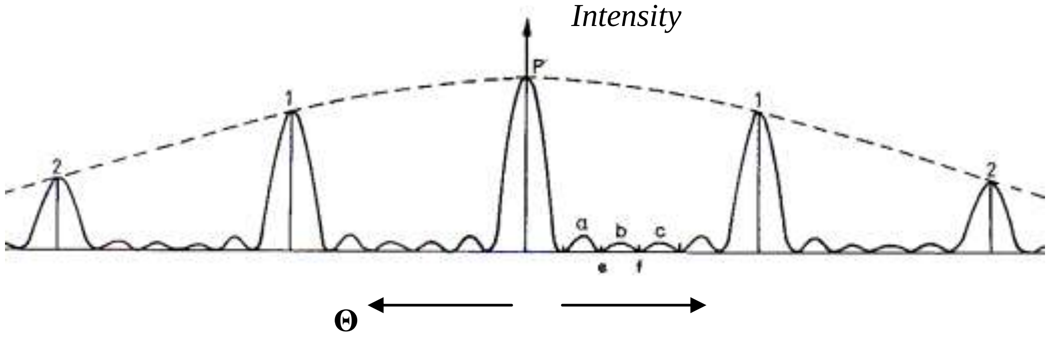


Fig. 3

The intensity distribution of the screen is shown in Fig.3, P corresponds to the position of the central maxima and 1, 2 etc. on the two sides of P represents the 1^{st}, 2^{nd} etc. principal maxima. *a, b, c,* etc. are secondary maxima and *e, f* etc. are the secondary minima. The intensity as well as the angular spacing of the secondary maxima and minima are so small in comparison to the principal maxima that they can not be observed. This results in uniform darkness between any two principal maxima.

I.5 *Sodium D Lines:*

The sodium doublet is responsible for the bright yellow light from a sodium lamp. The doublet arises from the $3p \rightarrow 3s$ transition in the sodium atom. The $3p$ level splits into two closely spaced levels with an energy spacing of 0.0021 eV. The splitting occurs due to the spin orbit effect. This can be crudely thought of as arising due to the internal magnetic field produced by the electron's circulation around the nucleus and

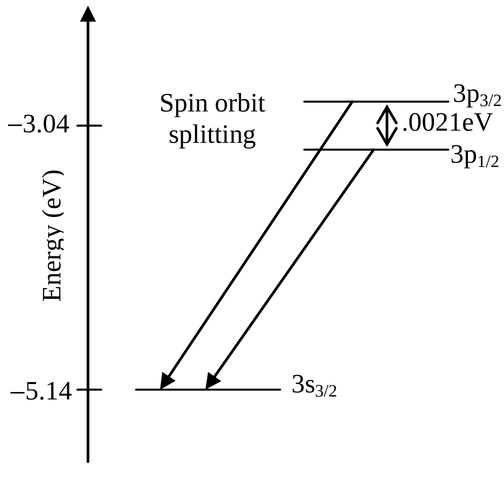


Fig. 4

the splitting takes place analogous to the Zeeman effect. Fig. 4 shows the *3p* and *3s* levels their splitting and the radiative transition that produces the sodium doublets or *D* lines.

II. Set-up and procedure:

PART A : *Calibration of diffraction grating:*

1. Adjust telescope for parallel rays i.e. focus telescope on the object at infinity. Here we can adjust telescope on an object which is at very large distance. Level the spectrometer and prism table on which grating is mounted using a spirit level. Fig. 5 schematically shows the arrangement of the grating and the spectrometer.

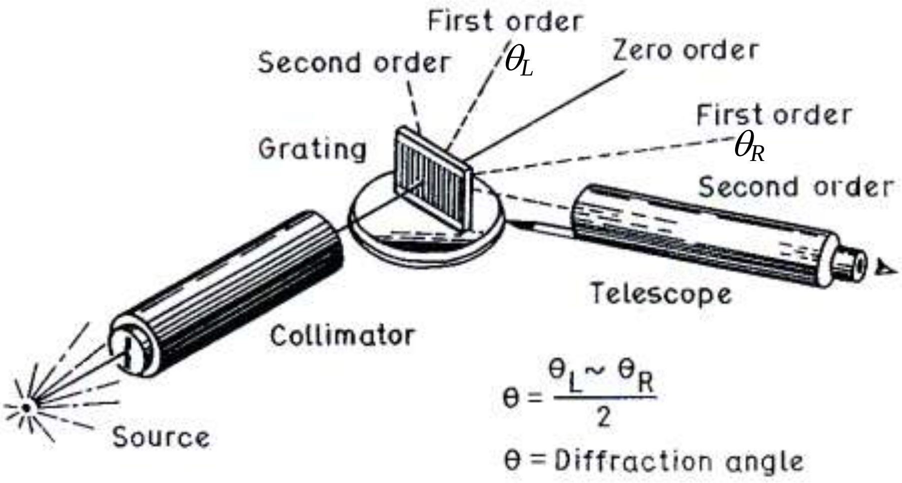


Fig. 5

2 Switch on the power supply for spectral lamp

2. Place the grating on the prism table such that the surface of the grating is approximately perpendicular to the collimator of the spectrometer (i.e., perpendicular to the incident slit falling on the grating). Fix the prism table in this position. With the Hg source observe first order spectrum on left hand side and right hand side. Measure the angle of diffraction of each line by rotating telescope so that cross-wire coincides with particular spectral line. Note down each measurement in the observation table 1. The diffraction angle is equal to difference between LHS and RHS observation divided by two for a particular spectral line. (See Fig. 5).

   The wavelengths of the main spectral lines of Hg in the visible region are given in Table 1.

PART B : *To measure the wavelength of second sodium line(D_2):*

3. Repeat steps 2 and 3 with sodium source. In first order spectrum of sodium measure the angular position θ_L of yellow 1 (*D1*) on the left side. Use the micrometer screw to turn the telescope to align the crosswire at the second yellow line (*D2*) and read its angular position θ_L.

4. Likewise measure θ_R on the RHS for *D1* and *D2*.

Precautions:

1. The experiment should be performed in a dark room.

2. Micrometer screw should be used for fine adjustment of the telescope. For fine adjustment the telescope should be first locked by means of the head screw.

3. The directions of rotation of the micrometer screw should be maintained otherwise the play in the micrometer spindle might lead to errors (also known as backlash error).

4. The spectral lamps (mercury source) attain their full illuminating power after being warmed up for about 5 minutes, so the observations should be taken after 5 minutes.

5. One of the essential precautions for the success of this experiment is to set the grating normal to the incident rays (see below). Small variation in the angle of incidence causes correspondingly large error in the angle of diffraction. If the exact normally is not achieved, one finds that the angles of diffraction measured on the left and on the right are not exactly equal. Read both the verniers to eliminate any errors due to non-coincidence of the center of the circular scale with the axis of rotation of the telescope or table.

Method to make light to fall normal to the grating surface:

a) First mount grating approximately normal to the collimator. See the slit through telescope and take reading from one side of vernier window. Note down the reading.

b) Add or subtract (whichever is convenient) 90° from reading taken in step a) and put telescope to this position. In this position telescope is approximately perpendicular to the collimator.

c) Now rotate prism table until the slit is visible on the cross-wire of the telescope. At this position the incident light from the collimator falls at an angle 45° with the plane of the grating. Note down this reading.

d) Next add or subtract 45° to step c) reading and rotate prism table so as to obtain this reading on the same window. In this situation light incident on the grating surface is perpendicular.

III. Exercises and Viva Questions:

1. What is a diffraction grating? How are they made? Name three different types of gratings.
2. Can a grating be used for studying spectra in the *UV* or infrared region? If so, what should be its characteristic? Can a prism be so used? What are the advantages of a grating over a prism?
3. The dispersion of a grating is defined as $D = \Delta\theta/\Delta\lambda$ where $\Delta\theta$ is the angular separation of the principal maxima of two lines whose wavelengths differ by $\Delta\lambda$. Show that the dispersion of a grating is $D = m/(d\cos\theta)$ at the mth order. Calculate D for the sodium doublet at the first order for your experiment.
4. The resolving power of a grating is defined as $R = \lambda_{avg}/\Delta\lambda$ where λ_{avg} is the mean wavelength and $\Delta\lambda$ the difference in the wavelengths of two spectral lines which can just be resolved into two lines. It can be shown that $R = Nm$, where N is the total number of rulings on the grating and m is the order at which the two spectral lines can be resolved. Calculate the number of rulings required to resolve the sodium doublet at the first order.
5. Use Bohr model for the frequency of light emitted in atomic transitions to calculate the wavelengths for the sodium doublet, using Fig. 4.
6. In the Hg spectrum which lines are prominent and which are weak? What could be the reason for variation in intensities of spectral lines?
7. What would be the advantages and disadvantages of looking at the second order spectra in this experiment?
8. What is the mechanism by which the emission spectrum is produced in the spectral lamps (Na or Hg)? Look up Ref. 2 for details.
9. What will happen if the incident light does not fall normally on the grating? Show that if ψ is the angle of incidence with respect to the normal to the grating, then the principal maxima occur at angles θ (w.r.t. the normal) such that $d(\sin\psi + \sin\theta) = m\lambda$. (This is problem 37.41 of Ref. 3).
10. Give examples of uses of gratings in Astronomy, Physics and Engineering.

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