Lecture series introduction:

Aspects of Symmetry

20th Century saw a paradigm shift in the approach toward Physical Sciences. In the classical physics, conservation laws were used as mere tools to compliment the dynamical methods of understanding physical system. Following, Noether's observations of deep connections between conservation laws and continuous symmetry transformations, today Symmetry principles are not only indispensable tools to understand dynamics but the very reason behind the existence of dynamics. In the Saturday discussion forum series, we shall explore some of the aspects of symmetry principles. We shall make a modest attempt to understand mathematical(group theoretical) basis of the Physics, as well as Physical basis behind the mathematics.

Permutation Symmetry and its far-reaching consequences

- Why can't we walk through the wall?
- How could a spin-independent Hamiltonian lead to spin-dependent splitting of energy levels?
- How to (anti)symmetrize just about anything --- Young tableaux, a powerful book-keeping method

"Innocuous" aspects of Rotation symmetry

- What is Angular Momentum after all?
- Spinning elementary tops: Zero-volume spinning objects that do not return to themselves upon rotation by 360 degree.
- Rotations in Real space --R2,R3 aka SO(2), SO(3), in complex space-C2,C3 aka SU(2), SU(3), Minkowski space aka SO(3,1),
- Quaternions -- generalized cousins of complex numbers, their relations to SO(3), SU(2).
- SU(2) and SO(3) are cousins and so are SO(3,1) and SL(2,C)

If the "Aspects of Symmetry" bus does not run out of fuel and if the driver does not crash of exhaustion and if there is at least one passenger on board, following is a random laundry list of beautiful things that we can discuss. I have very little hope of reaching this far though, but that will not deter me from an ambitious plan.

Not so innocuous aspects of Symmetry and its breaking

- Why should two electrons interact at all *aka* U(1) Gauge theory
- Why strong interactions are like a love-affair -- indifferent in proximity and infinitely possessive with growing distance -- Gauge theories of strong interactions SU(3)_c and Nobel prize 2004 (Gross, Wilczek, Politzer)
- "Origin" of Mass aka Spontaneous symmetry breaking (SSB) Nobel prize (2008, Nambu's work)
- SSB implies breakdown of ergodicity
- SSB implies phase transitions and hence Ferromagnetism, Superfluidity, Superconductivity
- Rotations in super-space (augmenting t,x,y,z with anticommuting grassmanian variables) and Supersymmetry.
- Why do we exist *aka* --Discrete symmetries, Parity, Charge Conjugation, Time reversal (Nobel 2008 again, Kobayashi and Maskawa's work)

Caution: Driver of the bus is still in the learning license phase (for last 15 years) and the tarrain is hilly and serpentine. Welcome aboard, but at on your own risk. Alternate drivers are welcome anytime. In all likelyhood this cannot be finished this semester and may spill over into next semester.

Lecture #2: Pauli's Exclusion Principle and Helium Atom

In the classical macroscopic world identical objects such as billiards balls can be distinguished by coloring or labeling them. Such a labeling does not alter the physics of Billiard balls in anyway. Quantum mechanical systems of identical particles such as electrons or photons are truly indistinguishable. This inability to distinguish physical situation under swapping of two electrons or photons has implications for the wavefunction of fermionic and bosonic systems and has dramatic consequences. In fact all of Chemistry and hence Biology is a consequence of the behavior of identical fermions under swapping. Such permutation symmetry implies a spin-statistics connection and hence the Pauli's exclusion principle. We will work through the classic example of the ground state and the excited state of Helium atom to explicitly see the how "Pauli repulsive force" does not allow two electrons in the same state. This calculation was among the first applications of Quantum Mechanics and is a beautiful example of a perturbation calculation that goes on to show that "esoteric quantum ideas" do faithfully reproduce experimental numbers.

Lecture # 3:

Theme : **The "Obvious" and the "Not-so-obvious" aspects of Rotation**

Abstract of the third lecture:

In the last lecture we saw that the permutation symmetry allows only two kinds of particles -- fermions and bosons, and this has far-reaching Physical, Chemical (and hence Biologoical) consequences. In this Saturday's talk we will begin our exploration of Rotational Symmetry.

What is the big deal about Rotation?

- *Classical Mechanics:* One of the most fundamental results in mechanics is a theorem due to Chasle which says that any general rigid body motion can be decomposed into a pure translation and pure rotation. Since rotation is "half" the physics of motion, it is very important to quantify what is meant by rotation.
- Quantum Mechanics, Relativity, Field Theory: Generator of rotation in 3-D euclidean space (aka angular momentum) is used to label the quantum state vectors. Relativity is a consequence of invariance under rotation in the Minkowski space (ct,x,y,z). Spin-half fermions like electrons are fundamental representations of rotation in an abstract 2 dimensional complex linear vector space. The very reason for the existence of various fundamental forces is a consequence of invariance under rotations in specific complex spaces.
- *Spin-Statistics Connection*: Behavior of fundamental particles under Permutation and Rotation is deeply connected! The Physics of spin-statistics connection has baffled the likes of Pauli, Schwinger, Feynman, Weinberg, Sudarshan.

We shall explore various ways of quantifying rotation and their inter-relations in this Saturday's discussion forum.

Lecture # 4:
Rotations: Connections with multiple Algebras

<u>*Recap of last Saturday's talk*:</u> We discussed that the Cartesian(xyz) coordinate system can be continuously rotated into different orientation (x'y'z') by multiplication with 3X3 Orthogonal matrix, parametrized in terms of three angles called Euler angles. We also found that there is a correspondence between this 3X3 real orthogonal matrix in Euclidean space and 2X2 Unitary matrix in 2 dimensional complex space. Pauli's spin matrices crop-up as a surprise in this purely classical and geometrical considerations. Why? The short answer is not easily comprehensible. We shall explore this connection in this saturday's talk.

<u>*This saturday's focus*</u>: We will explore the connection/correspondance between real 3X3 matrices and complex 2X2 matrices. It turns out that the rotation in 3-d real space can also be parametrized by an appropriate 4-d generalization of complex numbers called "Quaternions". Quaternions, conceived in 1826 by Willian Hamilton, are actually progenitors of the modern "vectors". We will delve a bit into the geometry of multiple algebras (quantities requiring more than one term for its specification). Geometry of quaternions will clarify how rotations in real and complex space are connected, why Pauli matrices crop in pure geometrical considerations and why electron when turned around by 360 degree, does not return to itself.

References: (AJP is available online)

1. Classical Mechanics by Goldstein (chap. 4)
2. Americal journal of Physics (AJP): Vol. 39, 1013
3. Americal journal of Physics (AJP): Vol.70, 958
4. The Road to Reality by Roger Penrose
5. Clifford Algebras and Spinors: Lounesto (Cambridge Uni. Press)

Lecture # 5:

Quaternions resolve "mysterious" fermions: Geometry meets Algebra

In our quest to understand rotations, we saw that for every 3-D real rotation matrix there are two complex matrices and hence the mapping is 1 to 2. This has a curious consequence that a rotation by 360 degree effected through 2-D complex matrices, makes the rotated object negative of itself which is counter-intuitive. This does not wreak havoc in the classical world, as 2-D complex matrices like in any complex analysis, are only mathematical constructs used to simplify manipulations in 3-D real space. However, Quantum Mechanics, admits intrinsically complex objects that live in 2-D complex space (all fermions), and these objects would have "ghostly" behavior of not returning to themselves upon rotations by 360 degree.

It turns out that in order to understand such mysterious behaviour, we need to expand the dimensionality of our number systems from 1-D real line, 2-D complex plane to higher dimensions. Interestingly, such expansion comes at the cost of sacrificing certain "sacrosanct" algebraic structures such as commutativity, associativity and divisibility. If one insists on divisibility, the only algebras possible are a list of numbers -- 1(real), 2(complex), 4(Quaternions -- non commutative) and 8 (Octonions --non-commutative & non-associative). We shall see how the geometry of the non-commutative algebra of Quaternions explains the intuitive and non-intuitive aspects of rotations in real and complex spaces. We shall also demonstrate using real objects in real space, how objects do not return to themselves upon rotation by 360 degree.

Lecture # 6:

Group Theoretical Basis of Symmetry Transformations

So far we have confined ourselves to Symmetry transformations performed in real and complex spaces and their inter-relations. We found that the branch of geometric algebra provides a very handy tool to intuitively understand transformations. Before we move on to rotations in Minkowski space and their relations to ordinary rotations, it is imperative that we understand the group theoretical basis of symmetry transformations.

In this Saturday's forum we shall explore the group theoretical notions of symmetry transformations, their representations, generators, Lie Algebras etc. We will come to appreciate that the conceptual building blocks of any physical theory -- spinors and tensors, are in fact understood as irreducible representations of some fundamental space-time symmetry transformations such as Rotations and Lorentz transformation. Noble Laureate Eugene Wigner was the pioneer of the space-time group symmetries.

References: (Any of the following is a good reference)

Group Theory in Physics -- Wu-Ki Tung (World Scientific)
Elements of Group Theory for Physicist -- A. W. Joshi (New Age International)
Quantum Field Theory - A Modern Introduction -- Michio Kaku (Oxford)
Quantum Field Theory -- Lewis Ryder (Cambridge Uni. Press)

Lecture # 7:

An unconventional but insightful derivation of Dirac equation

Schrodinger equation, being based on non-relativistic relation for energy and momentum is not suitable for describing quantum particles at relativistic speeds. A "naive relativistification" of the schroedinger equation is however beset with problems which Dirac solved brilliantly. Such, almost magical (and deceptively simple) solutions, however, leave much to our imagination. For instance, it turns out that in order to evade the problems with "naive relativistification" the equation is written in terms of four anti-commuting quantities(called alpha1,alpha2,alpha3 and beta) each of which is a 4X4 matrix. What is the physical significance of these alpha's and beta? What is their origin? These and host of other questions are better understood in an alternative and unconventional derivation of Dirac equation given by Gordan Baym in his beautiful book "Lectures on Quantum Mechanics". This derivation also demonstrates the power of symmetry transformations (Lorentz transformation in this case). The question as to "what Angular Momentum (and rotation) really is" will be put in to perspectives and related to its unlikely cousins (any guess?).

The previous lectures are not pre-requisites for this lecture.

Disclaimer: The speaker does not guarantee that you would walk out clear and resolved after the talk. In all likelihood you are likely to come out with "profound confusion" -- certainly a pre-requisite today if you want to buy any clarity tomorrow!