Simulation

- **Simulation** is the imitation of the operation of a real-world process or system over time. The act of simulating something first requires that a model be developed; this model represents the key characteristics or behaviors of the selected physical or abstract system or process. The model represents the system itself, whereas the simulation represents the operation of the system over time.
- Simulation is used in many contexts, such as simulation of technology for performance optimization, safety engineering, testing, training, education, and video games.

Simulation

- Simulation is also used with scientific modelling of natural systems or human systems to gain insight into their functioning.
- Simulation can be used to show the eventual real effects of alternative conditions and courses of action. Simulation is also used when the real system cannot be engaged, because it may not be accessible, or it may be dangerous or unacceptable to engage, or it is being designed but not yet built, or it may simply not exist .

Generation of Discrete Random variables

Let X be a random variable, having point probabilities:

$$P\left(X = x_j\right) = p_j, \quad j = 0,1,2,\ldots.. \sum_{j=0}^{\infty} p_j = 1$$

The value of X can be generated as:

Given a random number U (uniformly distributed on (0, 1)) set

$$X = x_0 \; if \;\; u < p_0$$

$$= x_1 \; if \quad p_0 \le u < p_0 + p_1$$

$$= - - - - - - - - - - - - -$$

$$= x_j \; if \sum_{i=0}^{j-1} p_i \le u < \sum_{i=0}^{j} p_i$$

$Setting:$

$$X = x_0 \;\; if \;\; u < F\left(x_0\right)$$

$$\quad = x_j \;\; if \;\; F\left(x_{j-1}\right) \le u < F\left(x_j\right),\; j = 1, 2, \cdots$$

Example:

Let X be a random variable, having point probabilities:

X	1	2	3	4
Probability	.2	.1	.4	.3

Generate the value of X.

$$X = \begin{cases} 1 & \text{for } u < .2 \\ 2 & \text{for } .2 \le u < .3 \\ 3 & \text{for } .3 \le u < .7 \\ 4 & \text{for } u \ge 0.7 \end{cases}$$

Random number

.427, .429, .571, .909, .035, .857, .947, .429, .608, .789,

.881, .566, .469, .119, .202, .076, .387, .792, .903, .061,

Find

$$\hat{P}(X = 1), \hat{P}(X = 2), \hat{P}(X = 3), \hat{P}(X = 4).$$

U	0.427	0.429	0.571	0.909	0.035	0.857	0.947	0.429	0.608	0.789
X	3	3	3	4	1	4	4	3	3	4
U	0.881	0.566	0.469	0.119	0.202	0.076	0.387	0.792	0.903	0.061
X	4	3	3	1	2	1	3	4	4	1

$$\hat{P}(X=1)=\frac{4}{20},\hat{P}(X=2)=\frac{1}{20}.$$

$$\hat{P}(X=3)=\frac{8}{20},\hat{P}(X=4)=\frac{7}{20}.$$

The Poisson Distribution

Poisson Distribution: A random variable X is said to have a Poisson distribution with mean λ, if its density function is given by:

$$\boxed{f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ for } x = 0, 1, 2, \cdots\cdots \qquad \lambda > 0.}$$

Generation of Poisson Random Variable:

$$f(x) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0, 1, \cdots\cdots$$

$Setting:$

$$X = 0 \ if \ \ u < F(0) \Rightarrow u < e^{-\lambda}$$

$$= j \, if \ \ F(j-1) \le u < F(j), \ j = 1, 2, \cdots$$

$$F(j) = \sum_{i=0}^{j} \frac{e^{-\lambda}\lambda^{i}}{i!}$$

9.4 Generation of Continuous Random variable

Let X be a on continuous random variable with the distribution function $F(x)$
Further let $U = F(x)$, then U is uniformly distributed over on the interval (0,1).

Proof:

Let $U = F(X)$, Then

$$F_U(u) = P(U \le u) = P(F(X) \le u)$$

$$= P(X \le F^{-1}(u)) = F(F^{-1}(u)) = u$$

$$\Rightarrow f_U(u) = 1, \quad 0 < u < 1$$

$\Rightarrow$ *U* is uniformly distributed between (0, 1)

Hence we have to solve the following problem:

$$u = F(x) \; for \quad x$$
$$i.e\, x = F^{-1}(u)$$

Example : Find the formula for generating value of X whose density function is

$$f(x) = \begin{cases} -x^3 & \text{for} -1 < x \leq 0 \\ \dfrac{x}{2} & \text{for} \;\; 0 < x \leq 1 \\ \dfrac{1}{2} & \text{for} \;\; 1 < x < 2 \\ 0 & otherwise \end{cases}$$

$$F(x) = \begin{cases} 0 & x \le -1 \\ \frac{1}{4}(1 - x^4) & \text{for } -1 < x \le 0 \\ \frac{1}{4}(1 + x^2) & \text{for } 0 < x \le 1 \\ \frac{1}{2}x & \text{for } 1 < x \le 2 \\ 1 & \text{for } x > 2 \end{cases}$$

For -1 < x < 0, 0 < $F(x)$ < 1/4
for 0 < x < 1, 1/4 < $F(x)$ < 1/2
for 1 < x < 2, 1/2 < $F(x)$ < 1

Solve
$u = F(x)$ for x
i.e. $x = F^{-1}(u)$

Then we have

$$Setting:$$

$$0 < u < 0.25 \Rightarrow -1 < x < 0$$

$$u = \frac{1}{4}\left(1 - x^4\right) \Rightarrow x = -\sqrt[4]{1-4u}$$

$$.25 < u < .5 \Rightarrow 0 < x < 1$$

$$u = \frac{1}{4}\left(1 + x^2\right) \Rightarrow x = \sqrt{4u-1}$$

$$.5 < u < 1 \Rightarrow 1 < x < 2$$

$$u = \frac{1}{2}x \Rightarrow x = 2u$$

Now we need to set the following:

$$If\ 0 < u < 0.25 \Rightarrow x = -\sqrt[4]{1-4u}$$

$$If\ .25 < u < .5 \Rightarrow x = \sqrt{4u-1}$$

$$If\ \ .5 < u < 1 \Rightarrow x = 2u$$

Generation of exponential random variable:

$$F(x) = 1 - e^{-\lambda x} \quad x > 0$$

$$set:$$

$$u = F(x) \Rightarrow u = 1 - e^{-\lambda x}$$

$$\Rightarrow -\lambda x = \ln(1-u)$$

$$\Rightarrow \; x = -\frac{1}{\lambda}\ln(1-u)$$

$$or, \quad x = -\frac{1}{\lambda}\ln(u)$$

Note that: If u is uniformly distributed on (0, 1) then 1 – u is also distributed on (0, 1), so we can replace 1 – u by u in above .

Application of Simulation

<u>Simulation of Queueing System:</u>

We take single-server queueing system with FCFS queue discipline, unlimited queue-length(or finite length) unlimited input source, known arrival and service pattern.

Aim: To find all performance measures (or characteristics) of the system.

- State of system changes only by an arrival or a departure
- A point in time at which the state of system changes is called **event.**

X : Inter-arrival Time

Y : Inter-Service Time

First we generate the random samples from *X* and *Y* using random numbers U_1 and U_2 and arrange them event wise.

Example: The management of a commercial bank plans to open a one teller window. Research study has projected the following distribution for inter-arrival time X

x (minutes):		5	6	7	8
$f_X(x)$	:	.15	.35	.35	.15

The service time (Y) distribution of the teller is:

y (minutes):		5	6	7
$f_Y(y)$	:	.25	.50	.25

Simulate the model for 35 minutes using the random numbers:

u_1 = 25, 37, 91, 10, 61

u_2 = 84, 01, 59, 40, 03

and estimate the characteristics of queueing system.

For inter-arrival time

$u_1 < 0.15$ then $x = 5$

$0.15 \leq u_1 < 0.50$ then $x = 6$

$0.50 \leq u_1 < 0.85$ then $x = 7$

$0.85 \leq u_1 < 1$ then $x = 8$

For service time

$u_2 < 0.25$ then $y = 5$

$0.25 \leq u_2 < 0.75$ then $y = 6$

$0.75 \leq u_2 < 1$ then $y = 7$

u_1	X (inter-arrival-time)	Arrival at time	u_2	Y (inter-service time	Departure at time
0.25	6	6	0.84	7	13
0.37	6	12	0.01	5	18
0.91	8	20	0.59	6	26
0.10	5	25	0.40	6	32
0.61	7	32	0.03	5	37

Event wise time	
6	A
12	A
13	D
18	D
20	A
25	A
26	D
32	D
32	A
37	D

Notation used

TBE: Time between events

TNA: Total no. of arrivals so far

TND: Total no. of departure so far

TNS: Total no in the system = TNA – TND

TNQ: Total no in the queue = TNS – 1 if TNS $\geq$ 1 and 0 if TNS = 0

TTS: Total time spent in the system between previous and present event = present TBE $\times$ previous TNS

TTQ: Total time spent in the queue between previous and present event = present TBE $\times$ previous TNQ

SS: Server Status (I: Idle or B: Busy)

IT: Idle time of server so far

SS = I if TNS = 0

=B if TNS $\geq$ 1

Present IT

= previous IT if previous server status is B

= previous IT + present TBE if previous server status is I

Estimation of Characteristic of the system

T $\rightarrow$ simulation time

$$\hat{\mathrm{L}} = \frac{\sum \mathrm{TTS}}{T}, \qquad \hat{L}_q = \frac{\sum \mathrm{TTQ}}{T}$$

$$\hat{W} = \frac{\sum \mathrm{TTS}}{TNA}$$

$$\hat{W}_q = \frac{\sum \mathrm{TTQ}}{TNA}$$

$$\hat{P}_0 = \frac{IT(\mathrm{End})}{T}$$

Tutorial Problem 7: The management of of a commercial bank plans to open a one teller window. Research study has projected the following distribution for inter-arrival time X

x (minutes):		5	6	7	8
$f_X(x)$	:	.15	.35	.35	.15

The service time (Y) distribution of the teller is:

y (minutes):		5	6	7
$f_Y(y)$	:	.25	.50	.25

Simulate the model for 35 minutes using the random numbers:

u_1 = 25, 37, 91, 10, 61

u_2 = 84, 01, 59, 40, 03

and estimate the characteristics of queueing system.

For inter-arrival time

$u_1 < 0.15$ then $x = 5$

$0.15 \leq u_1 < 0.50$ then $x = 6$

$0.50 \leq u_1 < 0.85$ then $x = 7$

$0.85 \leq u_1 < 1$ then $x = 8$

For service time

$u_2 < 0.25$ then $y = 5$

$0.25 \leq u_2 < 0.75$ then $y = 6$

$0.75 \leq u_2 < 1$ then $y = 7$

u_1	x	Arrival at Time	U_2	y	Departure at Time
0.25	6	6	0.84	7	13
0.37	6	12	0.01	5	18
0.91	8	20	0.59	6	26
0.10	5	25	0.40	6	32
0.61	7	32	0.03	5	37

Event wise time	
6	A
12	A
13	D
18	D
20	A
25	A
26	D
32	D
32	A
37	D

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
0	Start	-	0	0	0	0	0	0	I	0
6	A	6	1	0	1	0	0	0	B	6
12	A	6	2	0	2	1	6	0	B	6
13	D	1	2	1	1	0	2	1	B	6
18	D	5	2	2	0	0	5	0	I	6
20	A	2	3	2	1	0	0	0	B	8
25	A	5	4	2	2	1	5	0	B	8
26	D	1	4	3	1	0	2	1	B	8
32	D	6	4	4	0	0	6	0	I	8
32	A	0	5	4	1	0	0	0	B	8
35	End	3	5	4	1	0	3	0	B	8
							29	**2**		

$\mathrm{T} \rightarrow$ simulation time $= 35$ min.

$$\hat{\mathrm{L}} = \frac{\sum \mathrm{TTS}}{T} = \frac{29}{35} = .829,$$

$$\hat{L}_q = \frac{\sum \mathrm{TTQ}}{T} = \frac{2}{35} = .057$$

$$\hat{W} = \frac{\sum \mathrm{TTS}}{TNA} = \frac{29}{5} = 5.8$$

$$\hat{W}_q = \frac{\sum \mathrm{TTQ}}{TNA} = \frac{2}{5} = 0.4$$

$$\hat{P}_0 = \frac{IT(\mathrm{End})}{T} = \frac{8}{35} = 0.229 = \text{Proportion of idle time of the server}$$

Proportion of busy time of the server $= 1\text{-}\hat{P}_0$

Problem-2: The management of united commercial bank plans to open a one teller window. Research study has projected the following distribution for inter-arrival time X

x (minutes):	5	6	7	8
$f_X(x)$:	.15	.35	.35	.15

The service time (Y) distribution of the teller is:

y (minutes):	5	6	7
$f_Y(y)$:	.30	.350	.35

Simulate the model for 59 minutes using the random numbers:

25, 37, 91, 00, 61, 62, 80, 15, 23 for *X*.

and 84, 01, 59, 40, 03, 29, 50, 77, 32 for *Y*.

Also find the following: *L*, *Lq*, *W*, *Wq*, Proportion of busy time of the server.

Solution: problem-1

For inter-arrival time

$u_1 < 0.15$ then $x = 5$

$0.15 \leq u_1 < .50$ then $x = 6$

$0.50 \leq u_1 < .85$ then $x = 7$

$0.85 \leq u_1 <1$ then $x = 8$

For service time

$u_2 < 0.30$ then $y = 5$

$0.30 \leq u_2 < .65$ then $y = 6$

$0.65 \leq u_2 <1$ then $y = 7$

u_1	X (inter-arrival-time)	Arrival at time	u_2	Y (inter-service time	Departure at time
0.25	6	6	0.84	7	13
0.37	6	12	0.01	5	18
0.91	8	20	0.59	6	26
0.00	5	25	0.40	6	32
0.61	7	32	0.03	5	37
0.62	7	39	0.29	5	44
0.80	7	46	0.50	6	52
0.15	6	52	0.77	7	59
0.23	6	58	0.32	6	65

Event wise time			
6	A	37	D
12	A	39	A
13	D	44	D
18	D	46	A
20	A	52	D
25	A	52	A
26	D	58	A
32	D	59	D
32	A	64	D

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
0	Start	-	0	0	0	0	0	0	I	0
6	A	6	1	0	1	0	0	0	B	6
12	A	6	2	0	2	1	6	0	B	6
13	D	1	2	1	1	0	2	1	B	6
18	D	5	2	2	0	0	5	0	I	6
20	A	2	3	2	1	0	0	0	B	8
25	A	5	4	2	2	1	5	0	B	8
26	D	1	4	3	1	0	2	1	B	8
32	D	6	4	4	0	0	6	0	B	8
32	A	0	5	4	1	0	0	0	B	8

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
37	D	5	5	5	0	0	5	0	I	8
39	A	2	6	5	1	0	0	0	B	10
44	D	5	6	6	0	0	5	0	I	10
46	A	2	7	6	1	0	0	0	B	12
52	D	6	7	7	0	0	6	0	I	12
52	A	0	8	7	1	0	0	0	B	12
58	A	6	9	7	2	1	6	0	B	12
59 end	D	1	9	8	1	0	2	1	B	12
							50	3		

$$\mathrm{T} \rightarrow \text{ simulation time } = 59 \text{ min.}$$

$$\hat{\mathrm{L}} = \frac{\sum \mathrm{TTS}}{T} = \frac{50}{59}, \quad \hat{L}_q = \frac{\sum \mathrm{TTQ}}{T} = \frac{3}{59}$$

$$\hat{W} = \frac{\sum \mathrm{TTS}}{TNA} = \frac{49}{9}, \quad \hat{W}_q = \frac{\sum \mathrm{TTQ}}{TNA} = \frac{2}{9}$$

$$\hat{P}_0 = \frac{IT(\mathrm{End})}{T} = \frac{12}{59} = \text{Proportion of idle time of the server}$$

Proportion of busy time of the server $= 1\text{-}\hat{P}_0$

Tutorial Problem 8:

Assume that the inter-arrival time *X* and the service time *Y* are exponentially distributed with respective means 2 and 4/3 minutes. Simulate the model for 25minutes; starting with time zero. Use the following random numbers for (arrival): 20, 23, 86, 09, 92, 35, 38, 01, 24, 07, 50 (for departure): 21, 44, 27, 70, 73, 36, 59, 42, 85, 88 and 51. Hence find *L*, *Lq*, *W*, *Wq*, and Proportion of busy time of the server.

For inter-arrival time:

If $0 < u_1 < 1$ then $x = -2\ln(u_1)$

For inter-service time:

If $0 < u_2 < 1$ then $y = -(4/3)\ln(u_2)$

u_1	x	Arrival at time	u_2	y	Departure at time
0.20	3.21	3.21	0.21	2.08	5.29
0.23	2.93	6.14	0.44	1.09	7.25
0.86	0.30	6.44	0.27	1.74	8.99
0.09	4.81	11.25	0.70	0.47	11.72
0.92	0.16	11.41	0.73	0.41	12.13
0.35	2.09	13.50	0.36	1.35	14.85
0.38	1.93	15.43	0.59	0.70	16.13
0.01	9.21	24.64	0.42	1.15	25.79
0.24	2.85	27.49	0.85	0.21	27.71
0.07	5.31	32.80	0.88	0.17	32.97
0.50	1.38	34.18	0.51	0.89	35.07

Event wise time			
3.21	A	12.13	D
5.29	D	13.50	A
6.14	A	14.85	D
6.44	A	15.43	A
7.25	D	16.13	D
8.99	D	24.64	A
11.25	A	25.79	D
11.41	A	-	
11.72	D	-	

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
0	Start	-	0	0	0	0	0	0	I	0
3.21	A	3.21	1	0	1	0	0	0	B	3.21
5.29	D	2.08	1	1	0	0	2.08	0	I	3.21
6.14	A	0.85	2	1	1	0	0	0	B	4.06
6.44	A	0.30	3	1	2	1	0.30	0	B	4.06
7.25	D	0.81	3	2	1	0	1.62	0.81	B	4.06
8.99	D	1.74	3	3	0	0	1.74	0	I	4.06
11.25	A	2.28	4	3	1	0	0	0	B	6.34
11.41	A	0.16	5	3	2	1	0.16	0	B	6.34

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
11.72	D	0.31	5	4	1	0	0.62	0.31	B	6.34
12.13	D	0.41	5	5	0	0	0.41	0	I	6.34
13.50	A	1.37	6	5	1	0	0	0	B	7.71
14.85	D	1.35	6	6	0	0	1.35	0	I	7.71
15.43	A	0.58	7	6	1	0	0	0	B	8.29
16.13	D	0.70	7	7	0	0	0.70	0	I	8.29
24.64	A	8.51	8	7	1	0	0	0	B	16.80
25.00	End	0.36	8	7	1	0	0.36	0	I	16.80
							9.34	**1.12**		

$\mathrm{T} \rightarrow$ simulation time $= 25$ min.

$$\hat{\mathrm{L}} = \frac{\sum \mathrm{TTS}}{T} = \frac{9.34}{25},$$

$$\hat{L}_q = \frac{\sum \mathrm{TTQ}}{T} = \frac{1.12}{25},$$

$$\hat{W} = \frac{\sum \mathrm{TTS}}{TNA(\mathrm{End})} = \frac{9.34}{8},$$

$$\hat{W}_q = \frac{\sum \mathrm{TTQ}}{TNA(\mathrm{End})} = \frac{1.12}{8},$$

$$\hat{P}_0 = \frac{IT(\mathrm{End})}{T} = \frac{16.80}{25} = \text{Proportion of idle time of the server}$$

Proportion of busy time of the server $= 1 - \hat{P}_0$

Tutorial Problem 9:

Assume that in a queuing system with one server, FCFS queue discipline, unlimited queue length, unlimited Input source, the inter-arrival time and service time both are distributed exponentially with parameters 2 and 3. Simulate the system for 2.75 units of time and estimate The characteristics of the queuing system. Assume that Initially there are no customers in the system.

Random no. for inter-arrival time: 100, 973, 253, 376, 520

Random no. for service time: 692, 346, 140, 620, 650

For inter-arrival time:

If $0 < u_1 < 1$ then $x = -0.5\ln(u_1)$

For inter-service time:

If $0 < u_2 < 1$ then $y = -(1/3)\ln(u_2)$

u_1	x	Arrival at time	u_2	y	Departure at time
0.100	1.15	1.15	0.692	0.12	1.27
0.973	0.01	1.16	0.346	0.35	1.62
0.253	0.68	1.84	0.140	0.65	2.49
0.376	0.48	2.32	0.620	0.15	2.64
0.520	0.32	2.64	0.650	0.14	2.78

Event wise time	
1.15	A
1.16	A
1.27	D
1.62	D
1.84	A
2.32	A
2.49	D
2.64	D
2.64	A
2.78	D

Tut. Problem 11:

Assume that the inter arrival time X and the service time Y are exponentially distributed with mean 1 and 1/2 minutes respectively and with maximum allowed queue length to be 4. Simulate the model for 5 minutes by using the following random numbers

RN for X: 20, 23, 86, 09, 92

RN for Y: 01, 48, 75, 82, 69

Estimate all the parameters.

For inter-arrival time:

If $0 < u_1 < 1$ then $x = -\ln(u_1)$

For inter-service time:

If $0 < u_2 < 1$ then $y = -(0.5)\ln(u_2)$

u_1	x	Arrival at time	u_2	y	Departure at time
0.20	1.60	1.60	0.01	2.30	3.90
0.23	1.46	3.06	0.48	0.36	4.26
0.86	0.15	3.21	0.75	0.14	4.36
0.09	2.40	5.61	0.82	0.09	5.70
0.92	0.08	5.69	0.69	0.18	5.88

Event wise time	
1.60	A
3.06	A
3.21	A
3.90	D
4.26	D
4.36	D
5.61	A
5.69	A
5.70	D
5.88	D

Time	Event	TBE	TNA	TND	TNS	TNQ	TTS	TTQ	SS	IT
0	Start	-	0	0	0	0	0	0	I	0
1.60	A	1.60	1	0	1	0	0	0	B	1.60
3.06	A	1.46	2	0	2	1	1.46	0	B	1.60
3.21	A	0.15	3	0	3	2	0.30	0.15	B	1.60
3.90	D	0.69	3	1	2	1	2.07	1.38	B	1.60
4.26	D	0.36	3	2	1	0	0.72	0.36	B	1.60
4.36	D	0.10	3	3	0	0	0.10	0	I	1.60
5.00	END	0.64	**3**	3	0	0	0	0	I	2.24
							4.65			**2.24**

T $\rightarrow$ simulation time $= 5$ min.

$$\hat{\mathrm{L}} = \frac{\sum \mathrm{TTS}}{T} = \frac{4.65}{5},$$

$$\hat{L}_q = \frac{\sum \mathrm{TTQ}}{T} = \frac{1.89}{5},$$

$$\hat{W} = \frac{\sum \mathrm{TTS}}{TNA(\mathrm{End})} = \frac{4.65}{3},$$

$$\hat{W}_q = \frac{\sum \mathrm{TTQ}}{TNA(\mathrm{End})} = \frac{1.89}{3},$$

$$\hat{P}_0 = \frac{IT(\mathrm{End})}{T} = \frac{2.24}{5}$$ =Proportion of idle time of the server

Proportion of busy time of the server $= 1\text{-}\hat{P}_0$

Tut. Problem 1: The output of a firm's product over 50 typical days is given in the following table. Vehicles are available to deliver goods. Each vehicle makes one trip in a day and can deliver 105 units when fully loaded. As many as possible out of available for the day are loaded fully. Units in excess of total vehicle capacity are stored overnight and are included in the output of the next day. The number of vehicles available on number of days is given below for some 50 typical days. Simulate the model for 10 days, and hence find average daily output, average number of vehicles available, average number of units stored overnight.

Units Produced	No. of Days
500	3
550	7
600	10
650	20
700	10

Vehicles Available	No. of Days
5	8
6	18
7	10
8	8
9	6

Random numbers for units produced: 20, 23, 86, 09, 92, 35, 38, 01, 24, 07

Random numbers for vehicles available: 01, 48, 75, 82, 69, 36, 83, 10, 17, 04

Units Produced	Probability	Random No.	Vehicles available	Probability	Random No.
500	0.06	$0 < u < 0.06$	5	0.16	$0 < u < 0.16$
550	0.14	$0.06 \leq u < 0.20$	6	0.36	$0.16 \leq u < 0.52$
600	0.20	$0.20 \leq u < 0.40$	7	0.20	$0.52 \leq u < 0.72$
650	0.40	$0.40 \leq u < 0.80$	8	0.16	$0.72 \leq u < 0.88$
700	0.20	$0.80 \leq u < 1$	9	0.12	$0.88 \leq u < 1$

Day	RN	Unit produced	Total for delivery	RN	Vehicles available	Total capacity of VA C6*105	Unit stored overnight
1	.20	600	600	.01	5	525	75
2	.23	600	675	.48	6	630	45
3	.86	700	745	.75	8	840	0
4	.09	550	550	.82	8	840	0
5	.92	700	700	.69	7	735	0
6	.35	600	600	.36	6	630	0
7	.38	600	600	.83	8	840	0
8	.01	500	500	.10	5	525	0
9	.24	600	600	.17	6	630	0
10	.07	550	550	.04	5	525	25
		6000			64		145

Average daily output= 6000/10=600.

Average number of vehicles available=64/10=6.4.

Average number of units stored overnight=145/10=14.5.

Tut. Problem 4: A small retailer deals in a perishable commodity. The daily demand and the supply (in Kg.) are random variables. The past 500 days data show the following:

Supply	
Available (Kg)	No. of Days
10	40
20	50
30	190
40	150
50	70

Demand	
Demand (Kg)	No. of Days
10	50
20	110
30	200
40	100
50	40

The retailer buys the commodities at Rs. 20 per Kg. and sells at Rs. 30 per Kg. If any of the commodity remains at the end of the day it has no resale value and is a dead loss. Moreover, the loss on any unsatisfied demand is Rs. 8 per Kg.. Simulate for 6 days and find the net profit by using following random numbers:

For supply: 31, 63, 15, 07, 43, 81

For demand: 18, 84, 79, 32, 75, 27

Supply			Demand		
Available	Probability	Random No.	Demand	Probability	Random No.
10	.08	$0 \le u < .08$	10	.10	$0 \le u < .1$
20	.10	$.08 \le u < .18$	20	.22	$.1 \le u < .32$
30	.38	$.18 \le u < .56$	30	.40	$.32 \le u < .72$
40	.30	$.56 \le u < .86$	40	.20	$.72 \le u < .92$
50	.14	$.86 \le u < 1$	50	.08	$.92 \le u < 1$

Day	Random No.	Supply X	Random No.	Demand Y	Cost 20*X	Revenue 30*Y	Loss due to shortage	Profit
1	0.31	30	0.18	20	600	600	0	0
2	0.63	40	0.84	40	800	1200	0	400
3	0.15	20	0.79	40	400	600	160	40
4	0.07	10	0.32	30	200	300	160	-60
5	0.43	30	0.75	40	600	900	80	220
6	0.81	40	0.27	20	800	600	0	-200

Net profit = Rs. 400